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MOMENTUM SPECTRUM OF THE VAN ALLEN RADIATION

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If charged particles are injected at a certain point in a variable magnetic field, which has a number of scattering centers, the energy of the particles will increase (by "magnetic pumping") and at the same time the particles will diffuse away from the point of injection. A study of this process[1] shows that near the point of injection the momentum spectrum of the charged particles is $N = \text{const}\, p^{-n}$, with $n = 2 - \varphi$. Here φ is a small negative number (usually $-1 < \varphi < 0$) depending in an insensitive way on the properties of the scattering centers. In this way it is possible to explain the remarkable fact that the momentum spectrum of cosmic radiation obeys a power law, $N = \text{const}\, p^{-n}$, with n approximately constant (=2.6) from 10^{10} to 10^{19} ev/c.

It is of interest to see whether the Van Allen radiation can be explained by the same mechanism, which according to the description above is responsible for the acceleration of cosmic radiation. If this is true we should expect that the spectrum of the Van Allen radiation should obey the same power law as cosmic radiation.

The spectrum has recently been measured by Freden and White[2] who have plotted it as a function of the energy. If instead we plot it as a function of the momentum, we obtain the graph shown in Fig. 1. The straight line in the figure represents $N = \text{const}\, p^{-n}$, with $n = 2.6$ (obtained from measurements of cosmic radiation).

The fact that the measured points fall close to a straight line which has the same slope as for cosmic radiation, indicates that the theory may

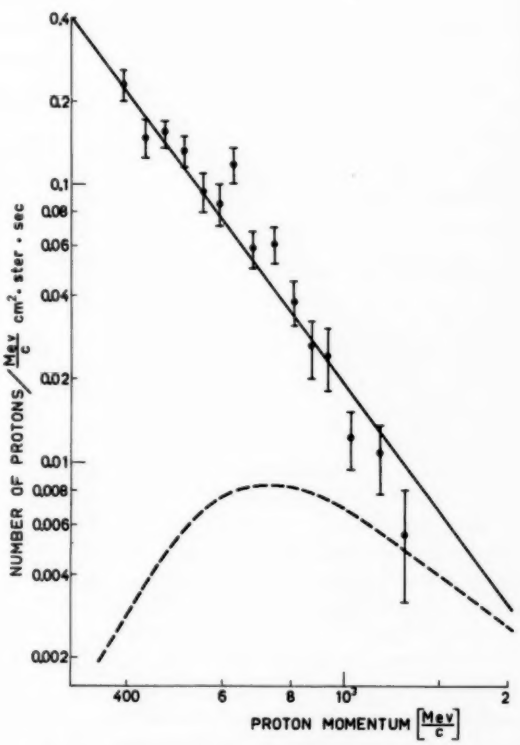

FIG. 1. Observational spectrum according to Freden and White, plotted with proton momentum as abscissa. Full line: $N = N_0 p^{-n}$ with $n = 2.6$ in agreement with cosmic-ray spectrum. Dashed line: Theoretically predicted spectrum [according to (1)] at some distance from the maximum of the Van Allen belt.

explain even the origin of the Van Allen radiation. This means that the particles are accelerated by variations in the earth's magnetic field. At the same time they diffuse away from the region of injection which defines the location of the maximum intensity.

The existence of two belts of radiation indicates that particles are injected in two different regions. It is natural to assume that the inner belt derives from the injection of particles from the ionosphere, possibly with some connection with the equatorial electro-jet.

The theory predicts that at a distance x from that line of force which passes through the point of injection, the spectrum should be

$$N = \frac{\text{const}}{p^{2-\varphi}} \exp\left(-\frac{x^2}{\kappa p^{1-\varphi}}\right), \qquad (1)$$

where κ is a constant.

Applying this to the Van Allen radiation, we should expect a spectrum as shown by the dashed curve in Fig. 1 with a cutoff at a momentum which depends on the distance from the point of injection. A reasonable measure of this distance would be the distance in the equatorial plane from that shell of magnetic field lines, at which the injection takes place. This prediction could be checked by future observations. For a more refined comparison the homogeneous field which is used in the model of the theory should be replaced by the earth's dipole field.

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OPTICAL PUMPING IN CRYSTALS

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An estimate was made of the expected population change in the ground state of a crystal due to optical pumping. Kastler's original pumping scheme[1] assumed that the ground state and an optically excited state are characterized by J values and have fairly pure m_J sublevels. Also, one needs a crystal with relatively long relaxation time T_1, as is the case for an S ground state.

Thus, Series and Taylor[2] have suggested optical pumping of Eu^{++} in CaF_2 or $SrCl_2$ and the observation of possible changes of magnetic-resonance absorption. The free Eu^{++} ion has intense lines in the 2300-2500 A region[3] ascribed to electric dipole transitions between the $4f^6 5d$ configuration and the $4f^7$ ground state $^8S_{7/2}$, presumably corresponding to the 3200 A and 2480 A bands in aqueous $EuCl_2$ solution observed by Butement[4] and the 3380 A band observed in a CaF_2:Eu^{++} crystal by Series and Taylor. Although correlation between the observed absorption spectrum and specific excited states is conjectural, we shall consider, as an example, optical pumping of the transition $J=7/2$ to $J=7/2$ between the ground state and the $^8P_{7/2}$ excited state.[5] One knows the transition probabilities KBI (B = coefficient of absorption and $K = 7/84,\ 12/84, \ldots$[6]) between a ground-state sublevel m_1 and an excited sublevel $m_2 = m_1 + 1$ due to irradiation with circularly polarized light of intensity I. Likewise known is the relative distribution of spontaneous emission from m_2 to $m_1 = m_2 - 1$, m_2, $m_2 + 1$. If one assumes a common relaxation time T_1 between all ground-state sublevels, and one writes the rate of change of sublevel populations due to both pumping and relaxation, one obtains in the steady state for the ground-state populations[7]:

$$N_m - N_{m-1} = (N_m - N_{m-1})_0 + pBIT_1N, \qquad (1)$$

where $(N_m - N_{m-1})_0$ is the Boltzmann equilibrium population difference, N is the paramagnetic-ion concentration, and p is a pumping coefficient listed in Table I. Similar orders of magnitude may be expected for transitions to the other optical states ($J = 5/2, 9/2$).

Series and Taylor estimate the oscillator strength of the 3380 A band in the CaF_2:Eu^{++} crystal comparable to Butement's values[4]

Table I. Pumping factors p between levels m_1 and m_1-1, for Eu^{++}.

m_1	m_1-1	p
7/2	5/2	1/860
5/2	3/2	1/570
3/2	1/2	1/1150
-5/2	-7/2	1/3400

$f=6\times10^{-3}$ (at 3200 A) and $f=3\times10^{-2}$ (at 2480 A) for $EuCl_2$ in solution. This estimate places $B=4\pi^2e^2f/mch\nu$ between 3×10^8 and 1.5×10^9 cm^2/erg sec. For a 500-watt 11 000-lumen xenon lamp, the radiation in all directions near 3300 A is $10^{-6}\,d\nu$ erg/sec. Thus, if an efficient optical system can concentrate 1/100 of that energy on a 1-cm^2 crystal surface and half the intensity is lost by polarization, $I=0.5\times10^{-8}$ erg/cm^2 is available. No appropriate europium samples were yet available in this laboratory for relaxation measurements; hence on the basis of measurements of Gd^{+++} at Bell Laboratories,[8] we take $T_1\approx10^{-3}$ sec. By taking the intermediate value for B, 9×10^8 cm^2/erg sec, we have

$$(N_{3/2}-N_{3/2})/N-(N_{3/2}-N_{3/2})_0/N=7.9\times10^{-8}.$$

This is only 1/1600 of $(N_{3/2}-N_{3/2})_0/N\approx h\nu/8kT=0.013$ at X band and 4.2°K.

For iron-group crystals, where the coupling between L and S is broken, the p factor can no longer be obtained from reference 6. However, the detailed work of Sugano and co-workers[9,10] on the red ruby lines R_1 and R_2 permits such a computation. The p factors of Table II are for magnetic field parallel to crystalline axis.[11] In all polarizations the necessarily simultaneous pumping of both lines is detrimental. The factor B to be used in Eq. (1) now depends on the polarization, e.g., $B_\sigma/B_\pi=10$ for the R_1 line.[11]

From spectrographic absorption measurements it was found that $B_\sigma\approx6\times10^3$ cm^2/erg sec for the R_2 line. Assuming that B_σ for R_1 is 5/3 as large,[11] we find that the strongest effect using both lines is obtained by pumping with σ^+ and σ^- radiation (i.e., natural light along crystalline axis), and observing the $3/2 \leftrightarrow 1/2$ paramagnetic absorption. The total effective p, referred to B_σ of R_2, is $(5/3)(-29+23)/440+(23-5)/216=2/33$. With $I=2\times10^{-8}$ erg/cm^2 available from the same xenon lamp (unpolarized light in red region) and $T_1=0.025$ sec,[12] $(N_{3/2}-N_{1/2})/N-(N_{3/2}-N_{1/2})_0/N\approx1.8\times10^{-7}$. This is less than 1/100 000 of $(N_{3/2}-N_{1/2})_0/N\approx h\nu/4kT=0.027$ for X band and 4°K, a hardly detectable change, mainly due to the very small B value. In experiments with π pumping of the red ruby lines, Wieder[13] detected no change in the microwave absorption. The light source, which used the fluorescence from the red lines themselves, provided an output of about 0.1 watt, thus giving approximately the same intensity over the linewidth as the above-mentioned xenon lamp.

From Gibson's[14] measurements of the green absorption band in ruby we find the value $B=3\times10^7$ cm^2/erg sec. Unfortunately, the sublevel transition probabilities have not been worked out, so the p factors cannot be computed. Assuming them to be of the same order of magnitude as those in Table II, one could change the paramagnetic population differences by one or two parts in a hundred for X band and 4.2°K, which would be detectable.

In the case of F centers in a KCl crystal, the ground and excited states are mixtures of s and p states with other states and information for computing p factors is not available. But even taking pessimistic p values, one can verify that this is a better prospect than the previously discussed substances. With $B=7\times10^{10}$ cm^2/erg sec (since $f=0.81$[15]), $I=10^{-8}$ erg/cm^2, $T_1=15$ sec at 4.2°K,[16] and $p=1.13\times10^{-5}$, inversion is reached; if $p=10^{-4}$, the original population difference $Nh\nu/2kT=0.053N$ has become $-0.46N$.[7] Thus the positive result of Lambe and Baker[13] for X-

Table II. Pumping factors p for ground-state transitions between m_S and m_S-1, for red ruby lines R_1 and R_2.

m_S	m_S-1	p for R_1 σ^+	σ^-	π	p for R_2 σ^+	σ^-	π
3/2	1/2	-29/440	23/440	3/44	23/216	-5/216	-1/12
1/2	-1/2	25/440	-25/440	0	-25/216	25/216	0
-1/2	-3/2	23/440	29/440	-3/44	5/216	-23/216	1/12

irradiated KCl can very plausibly be attributed to optical pumping. Although they used unpolarized light, a net pumping effect can result because, as in Table II, the p factors for σ^+ and σ^- need not cancel.

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[6]See, e.g., A. E. Ruark and H. C. Urey, Atoms, Molecules, and Quanta (McGraw-Hill Book Company, Inc., New York, 1930), p. 708, Eqs. (20)-(22).

[7]Equation (1), derived for small population changes, holds good even up to inversion of a two-level system, where the error is $h\nu/kT \approx 10\%$. For the extreme pumping in the last paragraph a more extended calculation was required.

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PURE NUCLEAR QUADRUPOLE RESONANCES IN PARAMAGNETIC IRON-GROUP HALIDES

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Although the number of compounds in which pure nuclear quadrupole resonances have been detected now runs into the hundreds, no resonance appears ever to have been reported in a paramagnetic substance. The observation, by Shulman and Jaccarino,[1] of the nuclear magnetic resonance of F^{19} in MnF_2 and some related paramagnetic compounds demonstrated that the nuclear resonance line width was not necessarily so broad as to preclude its detection. In fact, although the dipolar field of the Mn^{2+} ions in MnF_2 is very large and the hyperfine field due to the covalent bonding is even larger, both effects are swamped by the effective exchange field responsible for the antiferromagnetic ordering at 68°K. A detailed theoretical study by Moriya[2] of the relaxation processes in the antiferromagnetic and paramagnetic states anticipated the experimental findings. The same arguments might be expected to apply equally well in the pure quadrupole case. In spite of the large dipolar fields of the magnetic ions, the quadrupole resonances ought to be sufficiently exchange-narrowed to be observable if the substance has a large exchange energy (i.e., an ordering temperature greater than a few degrees K). Indeed, Jaccarino[3] has observed the quadrupole splitting of the nuclear magnetic resonance lines of Co^{59} in antiferromagnetic CoF_2.

A considerable number of iron-group chlorides and bromides are structurally identical to or very closely similar to the BiI_3 structure.[4] In this structure, the metal ions occur in layers between layers of close-packed halogens, such "sandwiches" being stacked to form the entire crystal. The layers of metal ions are thus separated by two halogen layers. Recent neutron diffraction work by the Oak Ridge group[5] has shown that these compounds become antiferromagnetic at appreciable temperatures, the ordering being with few exceptions ferromagnetic within the metal ion layer and with alternate layers arranged antiferromagnetically. Several diamagnetic compounds also utilize the BiI_3 structure, including AsI_3, SbI_3, and BiI_3 itself. The iodine nuclear quadrupole resonances in these compounds were reported some years ago[6]; the coupling constants appear to be consistent with the view that the bonding within the halogen-metal-halogen sandwich is largely covalent. The arsenic and antimony resonances have also been detected.[7] In consequence, it seemed reasonable to search for nuclear quadrupole resonances in some of the chlorides and bromides of the iron-group ele-

Table I. Nuclear quadrupole resonance frequencies of Cl^{35} and Br^{81} nuclei in iron-group halides.

Compound	νres at 24°C (Mc/sec)	Magnetic ordering temp. (°K)	Covalent character α^2
$TiCl_3$	7.39		0.27
$TiCl_2$	4.17		
VCl_3	9.40	30[a]	0.34
$CrCl_3$	12.812 12.848	17[b]	0.47
$CrBr_3$	85.73		0.53
$CrCl_2$	8.52	20[b]	

[a]Temperature of susceptibility maximum.
[b]Neutron diffraction value.

ments.

We have detected such resonances in polycrystalline samples of $TiCl_3$, $TiCl_2$, VCl_3, $CrCl_3$, $CrBr_3$, and $CrCl_2$. The observed Cl^{35} and Br^{81} resonance frequencies are listed in Table I. In all cases, the Cl^{37} and Br^{79} resonances were also detected to guarantee the isotopic assignment. The experimental apparatus consisted of a variety of superregenerative spectrometers; magnetic modulation and recording techniques were used in all cases, the resonances in $CrCl_3$ and $CrBr_3$ being the only ones observable with oscilloscope display. Figure 1 shows a portion of the Cl^{35} spectrum in $CrCl_3$. Two lines with an intensity ratio of 2:1 are observed, corresponding to crystallographically nonequivalent sites; the line widths are about 15 kc/sec. Attempts to record these resonances with a Pound-Watkins type regenerative spectrometer were unsuccessful, probably because of the poor performance of such spectrometers at higher rf power levels.

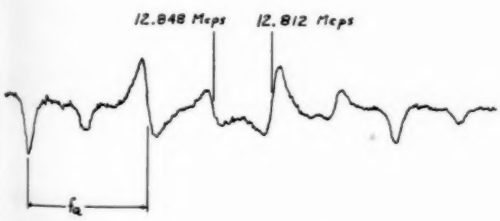

FIG. 1. Portion of superregenerative spectrometer tracing of Cl^{35} nuclear quadrupole resonance spectrum in $CrCl_3$. The quench frequency is 70 kc/sec.

Table I also lists the magnetic ordering temperature for those substances for which it is known. That resonances have been detected in some compounds for which ordering temperatures have not been determined suggests that the ordering temperatures of these compounds are also appreciable. In the case of $CrCl_3$, the resonance in the powdered sample was undetected at 4.2°K, but as the temperature was allowed to rise, the resonance appeared abruptly in the vicinity of 17°K. The large internal fields in the antiferromagnetic state give rise to a nuclear Zeeman energy which is at least comparable to the nuclear quadrupole energy, thereby rendering the resonance unobservable in the powdered sample.

The quadrupole resonance frequencies provide an estimate of the covalent character α^2 of the metal-halogen bond in these compounds. In the trihalides, the halogen coupling constant on a simple valence-bond model is given approximately by

$$eQq_{\text{obs}} = \tfrac{1}{2}\alpha^2 eQq_{zz}, \qquad (1)$$

where eQq_{zz} is the coupling constant for one unbalanced p-electron.

Values of α^2 calculated from (1) are also listed in Table I. They are in reasonable agreement with estimates based on the electronegativities of the atoms.[8] Considerations of s-hybridization and the weak bonding between adjacent halogen layers would modify these results somewhat. In the dihalides, eQq_{obs} depends very sensitively on the interbond angle θ at the halogen, which is about 89° in these compounds, so that the determination of α^2 is less reliable.

The covalent character may be related to the admixture coefficient λ of molecular orbital theory. Stevens[9] and Owen[10] have studied the effect of magnetic electron transfer on various ionic properties, and Owen[10] has estimated that the fractional importance of the anion $3d_\gamma$ orbitals in σ-bonding for Cr^{3+} in the chromium hexahydrate ion is given by $\lambda^2 \approx 0.7$. This may be compared with $\lambda^2 = 0.76$ for Cr^{3+} in $CrCl_3$ based on the Cl^{35} quadrupole coupling constant. Both results indicate considerable participation of the empty $3d_\gamma$ orbitals in σ-bonding. On the other hand, the fact that the F^{19} resonance in CrF_3 is virtually unshifted,[11] implies that the $3d_\epsilon{}^3$ configuration participates negligibly in covalent bonding.

The study of the magnetic resonance spectra of

both the cation and anion nuclei in a single crystal ought to be especially fruitful. In this way, a determination of the electric field gradient and magnetic hyperfine tensors at all nuclei in the crystal would be obtained. A complete analysis of covalent bonding structures could then be made.[12]

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CYCLOTRON RESONANCE IN $CdAs_2$

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Cyclotron resonance in nonmetals has been observed previously only in Ge,[1] Si,[2] InSb,[3] and recently in CdS.[4] Of these compounds only InSb was an intermetallic semiconductor, belonging to the III-V group of compounds. This Letter reports the first observation of cyclotron resonance in a II-V semiconductor, $CdAs_2$.

The crystals of $CdAs_2$ were grown from cadmium and arsenic of 99.999+% purity.[5] The crystals were spectrographically pure and x-ray analysis established them as single crystals of $CdAs_2$. Two disk-shaped n-type samples, 3 mm in diameter and about 0.6 mm thick, were used in the experiments reported here. The plane of the disk was perpendicular to the c axis for one sample and to the a axis for the second sample. All of the data were taken at frequencies between 20 000 Mc/sec and 24 000 Mc/sec and at temperatures below 2°K.

Figure 1 shows a typical plot of microwave absorption versus the magnetic field. Two absorption lines were resolved at all orientations. The higher field (higher effective mass) absorption line was identified with electrons by observation of microwave impact ionization and by excitation of carriers only from donor levels with filtered infrared radiation. When a Ge filter was used to eliminate light of energies sufficient to excite carriers across the 1.1-ev energy gap of $CdAs_2$, only the higher field (higher effective mass) resonance was observed at a diminished

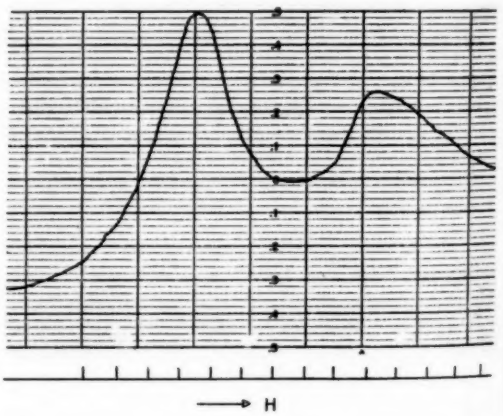

FIG. 1. Typical recording of power absorption as a function of magnetic field at a frequency of 20 835 Mc/sec and temperature below 2°K. The low-field line is identified with holes and the higher-field line with electrons.

intensity. The sample was known to be n-type from Hall effect measurements[6] and therefore it is quite certain that the higher effective-mass resonance was due to electrons. Further verification using circularly polarized microwave radiation will be carried out in the future. Linewidths were of the order of 700 gauss for the hole resonance and about 1000 gauss for the electron resonance in the better one of the two samples, but substantially greater in the second sample. The electron resonance had asymmetric shape as is evident in Fig. 1. The cause of this asymmetry is under investigation.

To determine whether plasma effects were present, measurements were made with the magnetic field in the plane of the sample disk and also with the field in a plane perpendicular to the surface of the disk. No significant differences in the spectra were found. Sharp variation tion in the intensity of the exciting light also had no effect on the spectra other than to change the intensity of the absorption lines. Over the very limited frequency range over which the experiments were conducted, the magnetic fields at which the resonances occurred scaled properly with frequency.

Figure 2 shows the variation in the effective mass with the magnetic field in the c-a plane as a function of the angle between the c axis and the magnetic field. The effective mass m^* measured in the cyclotron resonance experiments is given by

$$\frac{1}{(m^*)^2} = \frac{\cos^2\theta}{m_t^2} + \frac{\sin^2\theta}{m_t m_l}, \qquad (1)$$

where m_l and m_t are longitudinal and transverse mass parameters, and θ is the angle between the magnetic field and the longitudinal axis of the energy surface. The values of the effective-mass parameters obtained from a fit of the experimental data to Eq. (1) are given in Table I.

Table I. Effective-mass parameters of electrons and holes in $CdAs_2$ obtained from a fit of experimental data to Eq. (1).

	m_t	m_l
Electrons	$0.57m_0$	$0.16m_0$
Holes	$0.32m_0$	$0.11m_0$

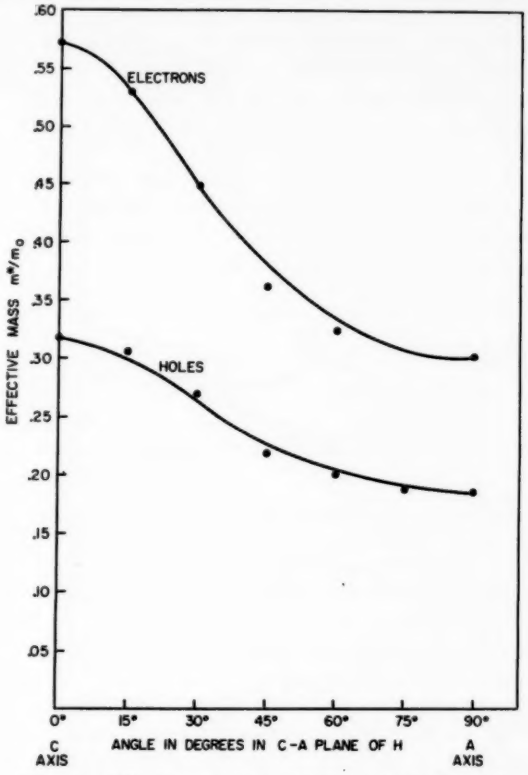

FIG. 2. Effective masses of holes and electrons in $CdAs_2$. The magnetic field is in the c-a plane at an angle θ from the c axis. Curves are theoretical, using Eq. (1), and points show experimental values.

The values of the effective-mass parameters for electrons agree well with the reported conductivity along the c direction, being four times greater than conductivity along the a direction.[7]

The spectrum obtained with the magnetic field rotated in the a-a plane was isotropic within the experimental accuracy, thereby indicating that the symmetry axis of the energy ellipsoids lies along the c axis. There was no indication of any structure in the absorption lines at any orientation and no other absorption lines were found in the range of effective masses $0.01m_0<m^*<0.75m_0$. All of the above experimental data suggest an energy band model for $CdAs_2$ consisting of single-energy ellipsoids for both holes and electrons with minima at $k=0$ and effective-mass parameters given in Table I.

Further experiments will be carried out at higher frequencies to attain better resolution, and to correlate the data with the Hall measure-

ments and with theoretical work on $CdAs_2$. The results will be reported later in a more complete paper on this material.

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OBSERVATION OF DIRECT TUNNELING IN GERMANIUM

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Holonyak et al.[1] have demonstrated experimentally the importance of phonons in tunneling from the (111) minimum of the conduction band of germanium to the (000) valence band maximum in accord with the theory of Keldysh.[2] In addition to the phonon process there is a "direct" process, treated by Keldysh[3] and Kane,[4] whereby an electron tunnels from the (000) valence band into the (000) conduction band minimum solely under the influence of the field without any additional perturbing mechanism. The "direct" process cannot occur in the forward bias region of a tunnel diode because there is no occupancy of the (000) minimum. However, in the reverse direction the filled states of the valence band may be made to coincide in energy with the empty states of the (000) minimum and "direct" tunneling will result. A very significant increase in tunneling current should be observed at this point, according to theory.

The transition from "indirect" to "direct" tunneling has been observed experimentally at 77°K in a germanium diode with 2×10^{18} carriers/cc on the n side and 8×10^{18} carriers/cc on the p side. The impurities were Sb and Ga. The diode had a barely discernible negative resistance in the forward direction as shown in Fig. 1(a).

Figure 1(b) shows the reverse characteristic

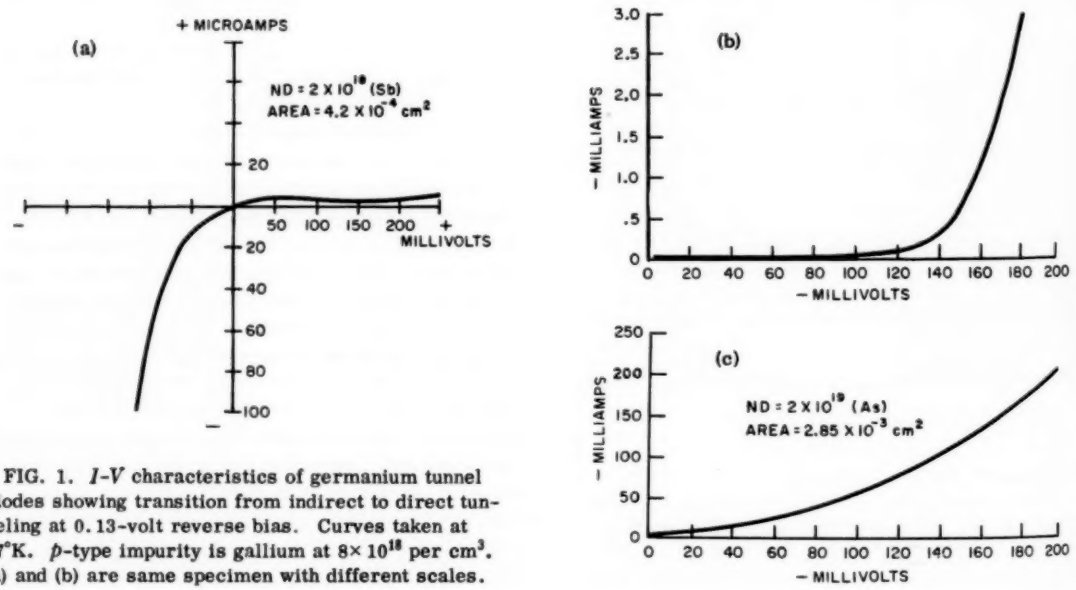

FIG. 1. I-V characteristics of germanium tunnel diodes showing transition from indirect to direct tunneling at 0.13-volt reverse bias. Curves taken at 77°K. p-type impurity is gallium at 8×10^{18} per cm^3. (a) and (b) are same specimen with different scales.

alone. The abrupt rise beginning at about 0.14 ev gives unmistakable evidence of the onset of direct transitions. The magneto-optical data of Zwerdling et al.[5] give 0.15 ev as the separation between the (111) and the (000) conduction band minima.

The tunneling calculations[3,4] were based on a two-band approximation and a constant electric field. The spin-orbit splitting in germanium invalidates the two-band approach and the junction field is linear rather than constant. In applying the formulas we attempt to correct for these errors by treating the field as a parameter to be determined by matching to the absolute current density. The theory gives

$$j=\frac{em_r\bar{E}_\perp}{9\hbar^3}\left\{eV_d-\bar{E}_\perp\left[1-\exp\left(\frac{-eV_d}{\bar{E}_\perp}\right)\right]\right\}\times\exp\left(\frac{-\pi m_r^{1/2}E_{000}^{3/2}}{2\hbar F}\right), \quad (1)$$

$$\bar{E}_\perp=\hbar F/2\pi m_r^{1/2}E_{000}^{1/2}, \quad (2)$$

where m_r is the reduced mass, $0.02m$, E_{000} is the band gap at $k=0$, F is the force on the electron, and V_d is the continuation of the p-side Fermi level measured from the bottom of the (000) minimum on the n side. (See Fig. 2.) $\bar{E}_\perp$ is the average kinetic energy of the tunneling carriers perpendicular to the junction field. Carriers with perpendicular energies much larger than $\bar{E}_\perp$ have greatly reduced tunneling probabilities. In deriving Eq. (1), ζ_p, the p-side Fermi level, was assumed much greater than $\bar{E}_\perp$. Temperature effects were also ignored. Neglecting the variation of F with V_d, Eq. (1) shows that the current increases quadratically for $V_d \ll \bar{E}_\perp$ and linearly for $V_d \gg \bar{E}_\perp$. Extrapolation of the linear slope back to $j=0$ gives a voltage intercept $+\bar{E}_\perp$. Calling ζ_n the Fermi level on the n-type side, we calculate that for $n=2\times10^{18}$ carriers/cc and $T=77°K$, $\zeta_n=0.006$ ev. The separation of the minima will then be given by

$$E_{000}-E_{111}=eV_0-\bar{E}_\perp+\zeta_n, \quad (3)$$

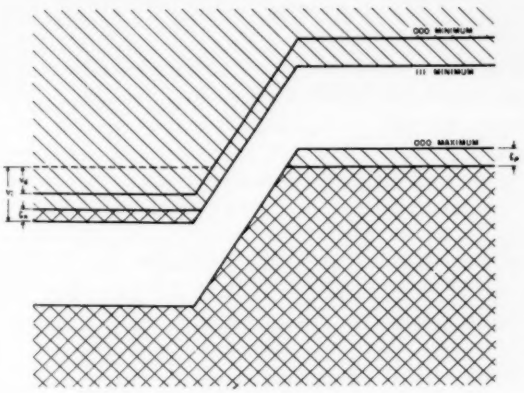

FIG. 2. Band edge contours in a tunnel diode with illustration of symbols.

where V_0 is the voltage intercept extrapolating linearly back to zero current. Taking a current density of 3.1 amp/cm² at 0.16 volt applied voltage from Fig. 1(b), we compute that $F=6\times10^5$ ev/cm, $\bar{E}_\perp=0.019$ ev. Using these values in Eq. (3), we find

$$E_{000}-E_{111}=0.13 \text{ ev}.$$

The discrepancy between this value and the magneto-optical value of 0.15 lies well outside the experimental error. It does not appear correct to ascribe the difference to an impurity-induced shift of the bands since measurements on an As-doped sample with 2×10^{19} carriers per cc on the n side as shown in Fig. 1(c), also give a separation of 0.13 ev. The latter value is less reliable, however.

In order to test the reasonableness of the assumed parameter F, we calculated the "effective" field for an abrupt junction with the measured carrier concentrations using parabolic energy bands. The effective field was defined to be the constant field which would give the same exponential attenuation factor as the variable field. The computed value was 3.5×10^5 ev/cm.

It is also of interest to check the magnitude of the indirect tunneling current, shown most clearly in Fig. 1(a). Using Keldysh's[2] result, we find

$$j=\frac{F^{3/2}(m_1m_2)^{3/2}(2n+1)(M^2V)}{32(2)^{3/4}\pi^{5/2}\hbar^{3/2}m_{rx}^{5/4}E_{111}^{7/4}}\left\{eV_i+\bar{E}_\perp\left[\exp\left(-\frac{(eV_i+\zeta_n)}{\bar{E}_\perp}\right)-\exp\left(\frac{-\zeta_n}{\bar{E}_\perp}\right)\right]\right\}\exp\left(\frac{-4(2m_{rx})^{1/2}E_{111}^{3/2}}{3\hbar F}\right), \quad (4)$$

$$\bar{E}_{\perp} = \hbar F/2(2m_{\gamma x})^{1/2} E_{111}^{1/2}, \qquad (5)$$

where m_1, m_2 are density of states masses, $m_{\gamma x}$ is the reduced mass in the field direction, E_{111} is the indirect energy gap, n is the average number of phonons, and M^2V is the square of the phonon matrix element times the volume. Again we have assumed $\zeta_p >> \bar{E}_{\perp}$. The right-hand side of Eq. (4) should be summed over the 4 conduction band minima but we assume that just one minimum is most favorably aligned for tunneling. For this minimum $m_{\gamma x}^{-1} = 0.08^{-1} + 0.04^{-1}$. We have determined M^2V from the magnitude of indirect optical absorption[6] to be

$$M^2V = 1.3\times10^{-47} \text{ erg}^2 \text{ cm}^3.$$

Using the above values together with the value of F determined for the direct case, we calculate a current density of 4×10^{-3} amp/cm^2 at 0.05 volt reverse bias, which is to be compared with the experimental value of 2×10^{-2} amp/cm^2. The factor of 5 discrepancy is within the uncertainty of the theoretical estimate. However, it is clear from Fig. 1(a) that the experimental curve shape does not agree with theory in the region between 0.05 ev and 0.12 ev since the slope increases rapidly with energy instead of becoming constant for $V_i >> \bar{E}_{\perp}$. ($\bar{E}_{\perp} = 0.015$ ev in this case.) The current increase is not rapid enough to be due to thermal excitation to the energy of the (000) minimum. We suggest that the increase is due to the impurity admixture of states in the (111) and (000) minima. In perturbation terminology, the particle tunnels into a virtual state in the (000) minimum and is then impurity-scattered into the (111) minimum. The rapid increase in current with voltage would then be due in part to the decrease of the energy denominator.

Holonyak et al.[1] have observed phonon-assisted tunneling in Sb-doped junctions of germanium but not in P- and As-doped junctions. A possible interpretation is that impurity scattering dominates phonon scattering for the latter two impurities. Since the k-vector change is a large one, a variation with the specific impurity is to be expected. Their results are also consistent with the observation that Sb has the smallest ionization energy of the column V impurities, hence the wave function is least concentrated at the core. A good test of this interpretation would be a comparison of direct and indirect tunneling currents in the reverse characteristic for Sb, As, and P. The latter two should have the smaller ratio.

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OPTICAL DETECTION OF PARAMAGNETIC RESONANCE SATURATION IN RUBY*

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(Received June 24, 1959; revised manuscript received October 12, 1959)

In order to extend the technique of optical pumping to solids, it is convenient to employ a solid system with sharp optical lines.[1] One such material is ruby which consists of Cr^{+++} ions in a host crystal of Al_2O_3. The optical spectrum of ruby has been studied extensively[2-6] and includes two sharp lines denoted by R_1 and R_2 which appear in both absorption and emission. The Zeeman effect of these lines was also observed and has recently been investigated both theoretically and experimentally in detail.[7,8] The results of the above-mentioned work are shown in the high-field transition diagrams of Fig. 1. The expected zero-field intensities were obtained by assuming[9] that the optical matrix elements are magnetic-field independent and adding the appropriate values for the degenerate levels. It is apparent that many possibilities exist for preferentially depopulating one of the sublevels of the ground state. For example, a mixture of R_1 and R_2 radiation polarized with the electric vector along the optic axis can be expected to depopulate pref-

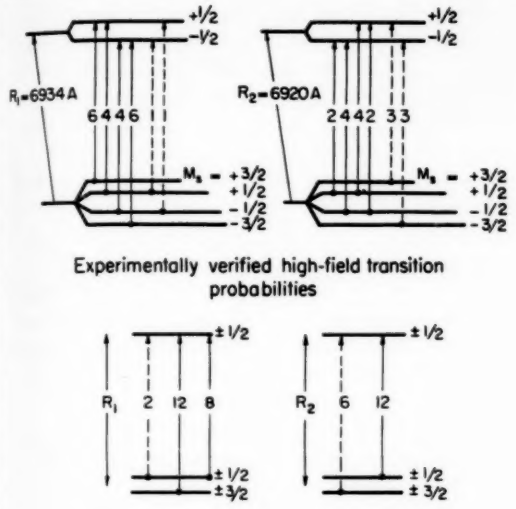

FIG. 1. Transition probabilities for the R lines in ruby. The dashed and solid lines refer to the cases where the exciting light has its electric vector parallel and perpendicular, respectively, to the optic axis. The external magnetic field is parallel to the optic axis.

erentially the $\pm\frac{3}{2}$ level.

The present experiments were performed at about 2°K (below the lambda point of liquid helium) to lengthen spin lattice relaxation times and to eliminate bubbles in the optical path. At this temperature, the R absorption widths are about 0.6 cm^{-1} for pink ruby.[8] Although broad spectrum sources emit about 0.01 watt of light energy in this bandwidth at 6900 A, there is an associated extraneous background light even with narrow-band optical filters. This background light causes undesirable heating of the sample and excludes optical detection methods.

It is apparent that a source of "resonance radiation" can overcome both of the above difficulties. In the case of ruby, the R lines appear as fluorescent lines upon absorption of light by the Cr^{+++} ions in the two broad bands denoted in Fig. 2 by U and Y. The relaxation from the broad excited states to the sharp levels denoted[7] by ^2E is radiationless, and the decay to the ground state has a lifetime of milliseconds.[10] This photoluminescent effect was utilized in the design of a solid state light source which emits a total of about 0.1 watt in each of the R lines. The details of this source have been described elsewhere.[11]

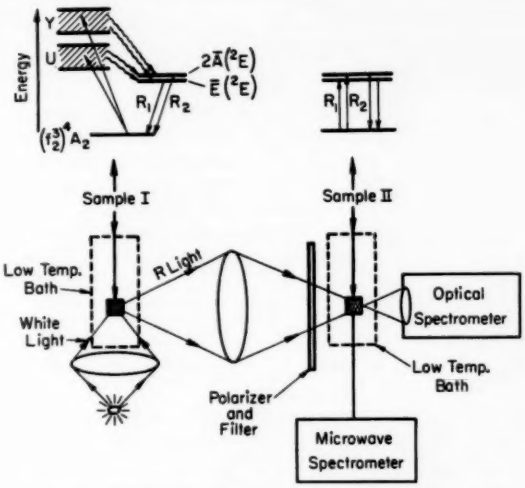

FIG. 2. Schematic diagram of the microwave optical experiments in ruby showing the different optical cycles in the two samples.

The experimental arrangement is shown schematically in Fig. 2. Sample I, which is utilized as the source, is irradiated with light from a tungsten source and undergoes the photoluminescent cycle mentioned above. The R fluorescence from sample I is collected, polarized, and focussed on sample II, which is in the cavity of a microwave spectrometer. Due to the temperature dependence of the R frequency,[3,8] it is necessary to keep sample I at liquid nitrogen temperature or cooler so that its R emission frequencies overlap the R absorption frequencies of sample II at liquid helium temperature.[11] In general, optical pumping disturbs the equilibrium populations of the sublevels of the ground state and can be detected by using ordinary paramagnetic resonance techniques. In this case, the R light is chopped, and a corresponding time-varying microwave absorption signal can be synchronously detected. The expected signal-to-noise ratio using this method is marginal with the present apparatus, and although the experiment was tried, the absence of a signal cannot be taken too seriously.

However, in order to verify that preferential optical depopulation of sublevels can occur in ruby at low magnetic fields, another experiment utilizing the same apparatus was attempted. In this case, the low-temperature alignment of the ground state is altered by microwave saturation. This increases the number of centers in the

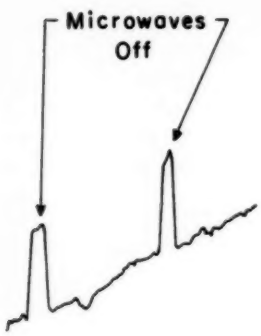

FIG. 3. Recorder tracing of a microwave-induced change of transmitted R_2 light polarized with its electric vector perpendicular to the optic axis.

$m = \pm \frac{1}{2}$ levels of the ground state and should change the R absorption, provided that the rate of optical transitions from the $m = \pm \frac{1}{2}$ levels differs from that of the $m = \pm \frac{3}{2}$ levels. This was actually found to be the case, as can be seen in Fig. 3. The intensity of transmitted R_2 light was displayed on a recorder, and the microwaves were manually turned on and off. The signal corresponds to about a one percent change in transmitted light for complete microwave saturation. The sloping background is due to a drift in the detection apparatus. Signals from R_2 light polarized with its electric vector parallel or perpendicular to the optic axis showed increasing or decreasing light transmission, respectively. This is in qualitative agreement with the transition probabilities shown in Fig. 1. A preliminary study of the amplitudes and signs of the signals indicates that the intensity distribution of light in the source is also contributing to the signal. This may be partially due to a slight frequency shift between the source emission line and the sample absorption line.[11] With the difference in rates of optical absorption now established, it may be possible to enhance this effect and to detect optical pumping directly. These points are being pursued further and will be reported on later.

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FORMATION OF NEGATIVE IONS IN CO BY ELECTRON CAPTURE FROM FAST HYDROGEN ATOMS

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(Received October 23, 1959)

A recent precise measurement of the total cross section for electron loss by a hydrogen atom in collision with a hydrogen molecule has revealed structure in that cross section.[1] A proposal has been made, very tentatively, that enhancement of the cross section occurs at certain energies because at such energies the atomic electron may be captured by the target molecule as well as go into an unbound state. On the basis of the very sketchy evidence available a mechanism for this process was also developed. It was proposed that in collisions leading to the formation of $H^- + H$ the electron first makes a transition into the continuum as the hydrogen atom approaches the molecule, and attains about 0.25 ev of kinetic energy in the proton-electron center-of-mass system. The angular distribution is to be peaked strongly forward. To the molecule then this electron is seen as a free electron which can be captured into a state of

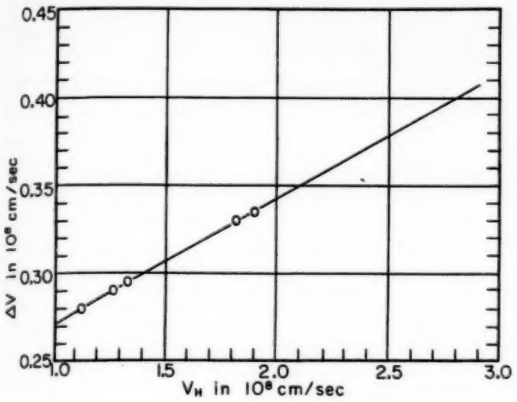

FIG. 1. Velocity of the free electron in the hydrogen atom system as a function of the hydrogen atom velocity in the laboratory. The points are taken from $H+H_2$ data.

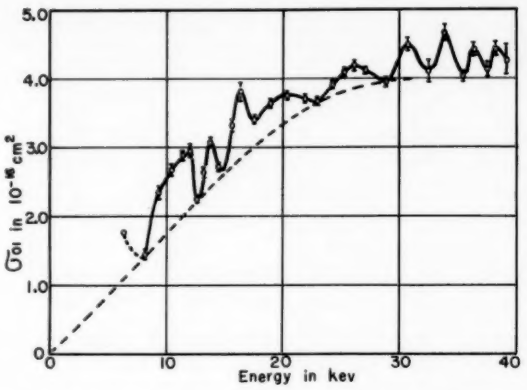

FIG. 2. Electron loss cross section for hydrogen atoms in CO. At the lowest energy a measurement of $\sigma_{01}+\sigma_{0-1}$, but not $\sigma_{01}-\sigma_{0-1}$, is available.

H_2^- in accordance with the Franck-Condon principle provided that its energy in the lab system, $E_e=mV_e^2/2$, is proper. On the basis of this picture it was found necessary to assume that the velocity the electron acquires in the H atom system, ΔV, is a slowly varying function of the velocity of the atom, V_H (Fig. 1). Thus, if

$$V_e = V_H + \Delta V, \tag{1}$$

ΔV is found to be linear with V_H. In that case the energy at threshold of the electrons stripped from H,

$$E_e = (m/M)E_H + E_0 + 2[(m/M)E_0E_H]^{1/2}, \tag{2}$$

coincides with the thresholds found by Schulz[2] and by Khvostenko and Dukel'skii[3] for free electrons in H_2. Here E_H is the kinetic energy of the atoms in the lab system and E_0 is the kinetic energy of the electron in the moving atomic system after it has been stripped.

To test these ideas we have measured the loss cross section for hydrogen in argon and in carbon monoxide. In the first case we find a smooth cross section with no evidence of structure. In the second we have found a multitude of very large peaks reaching more than 10^{-16} cm² above the smooth background (Fig. 2). If we treat the thresholds again exactly as the H_2 case, in fact take the increments ΔV directly from Fig. 1, we

Table I. Comparison of electron energies and appearance potentials.

H energy kev	Electron energy ev	Appearance potentials ev	
8.10	6.59		
12.55	9.71	9.66	$C(^3P)+O^-$
14.45	11.03	10.92	$C(^1D)+O^-$
16.6	12.5	12.34	$C(^1S)+O^-$
23.50	17.15	17.11	C^*+O^-
28.77	20.65	20.93	$C^+(^2P)+O^-+e$
32.4	23.30	23.40[a], 23.63[b]	C^-+O^++e
35.40	25.25		
36.9	26.12	26.26	$C^+(^4P)+O^-+e$

[a] Petrocelli, reference 6.
[b] Lagegren, reference 5.

find a very close agreement between the resulting electron energies and the expected appearance potentials of O^- and C^-. In Table I we have listed these energies and the appearance potentials for negative ions derived from CO if the dissociation energy of CO is taken as 11.11 ev and the affinity of O as 1.45 ev.[4] For $C^- + O^+$ we have listed values taken from the theses of Lagegren[5] and Petrocelli[6] separately. In CO, as in H_2, we find at least one very broad peak below the first one that appears from free electron capture. In the present case the ion formed would have to be CO^- if our ideas are correct. A mass spectrographic analysis is under way to elucidate these processes further.

This work was supported in part by the Office of Naval Research and the U. S. Army Office of Ordnance Research.

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DIPOLE STATE IN NUCLEI*

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In order to explain the unexpectedly high (γ, p) cross section in heavy nuclei, various authors[1,2] have proposed that these protons arise mainly from a direct process. The close relationship of this process to the shell model and optical model has been elucidated by Wilkinson,[3] who points out that the initial state of the nucleus is quite well described by the shell model. The proton involved in the direct process can then be considered as being initially in an eigenstate in the shell-model well. Upon absorption of the dipole gamma ray, the proton makes a transition to either a bound level in the well or one in the continuum. Because this state is not stationary, it is given a width $\Gamma + 2W$, where W is the absorption in the optical-model well at the relevant excitation and describes the absorption of the single-particle excitation into compound states and Γ is the width for escape. The proportion of fast protons that escapes is then $\Gamma/(\Gamma + 2W)$. The picture is very appealing, in that it produces the observed order of magnitude of fast particles, which is several orders of magnitude greater than the statistical description predicts. The relation of this description to one in terms of compound states of the system has been given in detail.[4] In reference 4 it is made clear that the highly excited levels discussed by Wilkinson are really combinations of thousands or millions of compound states which, however, act coherently as a single-particle state for some processes. We will refer to these groups of compound states as single-particle excitations.

The positions of the single-particle excitations can be found directly through other processes. Recently, (d, p) experiments using poor resolution[5,6] have determined the positions of single-particle excitations lying between zero bombarding energy and the binding energy of the last neutron. The spacings of these excitations seem to be in sharp conflict with those required by Wilkinson. For example, in Ti^{49}, the $f_{7/2}$ and $g_{9/2}$ levels are only about 4 Mev apart. However, just the transition between these two levels is an appreciable part of the giant dipole resonance in this nucleus, which comes at an energy of about 15 Mev. It is true, of course, that one must add a pairing energy to the 4 Mev before making the comparison, because in the absorption of the gamma ray a pair is generally broken. However, this is only one or two Mev. It seems, therefore, that the transition between single-particle excitations should occur at an energy of only about half that of the giant dipole resonance.

We should like to point out in this note that these two energies cannot be compared directly, since, in the dipole absorption, a hole is formed in the nucleus. Since the process is a dipole one, the excited particle and hole are strongly correlated in angle; i.e., their angular momen-

tum must be coupled to form a 1- state, assuming the original nucleus to be in a 0+ state. Because many particle-hole states can be formed, and because these states are almost degenerate in energy, the particle-hole interaction can have a profound effect in redistributing dipole transition strength.

We shall demonstrate these effects by using a schematic model, the mathematics of which is suggested by the Copenhagen work on pairing interactions.[7] Major numerical approximations are made in going from the actual situation to this rough schematic model. The model does, however, exhibit how coherent effects are able to push the dipole transitions to much higher energies than one would, at first sight, think possible.

We consider first protons in a potential well, and will indicate the extension to the case of protons and neutrons in a nucleus later. The main contributions to the absorption come from closed shells. We shall, therefore, specialize our discussion to nuclei with double closed shells, neglecting the influence of the few valence nucleons. We neglect spin, and consider only transitions from l to $l+1$. Choosing our "vacuum" as the initial nucleus, we see that the gamma ray creates a particle-hole pair through the process shown in Fig. 1. With axes oriented so that the Hamiltonian describing the interaction with radiation is

$$H_I = e(2\pi\hbar\omega)Z, \tag{1}$$

the particle-hole state formed in absorption of the gamma ray is

$$\varphi_i(\vec{r}_p, \vec{r}_h) = (-)^{l_i}[Y_{l_i}(\theta_h, \varphi_h)Y_{l_i+1}(\theta_p, \varphi_p)]^1_0 R_{l_i}(r_h)R_{l_i+1}(r_p), \tag{2}$$

where

$$[Y_{l_i}Y_{l_i+1}]^1_0 = \sum_m C(l_i, l_i+1, 1; m, -m, 0)Y_{l_i}{}^m Y_{l_i+1}{}^{-m}.$$

The lower suffixes p and h refer to particle and hole, the R's are radial wave functions, and the C's are Clebsch-Gordan coefficients. The phase factor $(-1)^{l_i}$ is put in for convenience to make all quantities positive. In medium and heavy nuclei, a large number of particle-hole states can be formed. The particle-hole interaction will mix these, and to find the perturbed eigenstates, we must solve the secular equation. In realistic situations, this is quite complicated, e.g., see Elliott and Flowers[8] where this is carried out for O^{16}. We shall, therefore, make several approximations which give us a schematic model where we can display the qualitative features explicitly. We shall return later to a discussion of the approximations. First, we shall use zero-range forces, i.e., we take the particle-hole interaction to be

$$V(\vec{r}_p - \vec{r}_h) = V_0\delta(\vec{r}_p - \vec{r}_h),$$

with V_0 positive. (If the particle-particle force is attractive, the particle-hole one is repulsive.) Now when Y_{l_i} and Y_{l_i+1} have the same argument,

$$[Y_{l_i}Y_{l_i+1}]^1_0 = (-)^{l_i}[(l_i+1)/4\pi]^{1/2}Y_1{}^0, \tag{3}$$

so that the diagonal elements of our secular matrix are

$$\epsilon_i + (l_i+1)(V_0/4\pi)\int_0^\infty R_{l_i}{}^2 R_{l_i+1}{}^2 r^2 dr,$$

where the ϵ_i are the unperturbed energies, i.e., the energies of the dipole excitations Wilkinson is considering. The off-diagonal elements are

$$(l_i+1)^{1/2}(l_j+1)^{1/2}(V_0/4\pi)\int_0^\infty R_{l_i}R_{l_i+1}R_{l_j}R_{l_j+1}r^2 dr.$$

We next make the further approximation of set-

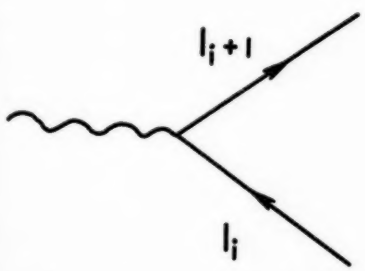

FIG. 1. Diagram representing the process by which a gamma ray creates a particle-hole pair.

ting the radial integrals equal, i.e., we set

$$(V_0/4\pi)\int_0^\infty R_{l_i}{}^2 R_{l_i+1}{}^2 r^2 dr = (V_0/4\pi)\int_0^\infty R_{l_i} R_{l_i+1} R_{l_j} R_{l_j+1} r^2 dr = G, \tag{4}$$

giving us a secular equation

$$0 = \begin{vmatrix} \epsilon_1 + (l_1+1)G - \lambda & (l_1+1)^{1/2}(l_2+1)^{1/2}G & (l_1+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_2+1)^{1/2}G & \epsilon_2 + (l_2+1)G - \lambda & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_3+1)^{1/2}G & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \epsilon_3 + (l_3+1)G - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}$$

By collecting powers of $(\epsilon_i - \lambda)$ one can easily show that this is equivalent to the equation

$$\prod_{i=1}^{n}(\epsilon_i - \lambda) + \sum_{j=1}^{n}\prod_{i=1(i\neq j)}^{n}(\epsilon_i - \lambda)(l_j+1)G = 0, \tag{6}$$

which in turn is equivalent to

$$\sum_{j=1}^{n}(l_j+1)/(\lambda - \epsilon_j) = 1/G. \tag{7}$$

The solutions of Eq. (7) can be obtained by plotting the left- and right-hand sides graphically as in Fig. 2, where the $\times$'s show the solutions λ_n. It is seen that the uppermost eigenvalue is pushed up a large amount. Let us now ask what happens when the ϵ_i become degenerate. Then, it easily is found that the solutions λ_i are

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \epsilon, \qquad \lambda_n = \epsilon + \sum_{i=1}^{n}(l_i+1)G, \tag{8}$$

where ϵ is the common value of the ϵ_i. Denoting

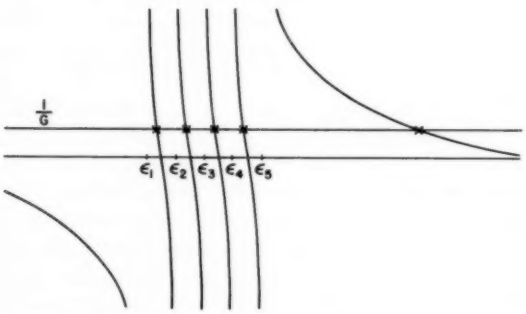

FIG. 2. Graphical solution of Eq. (7).

the perturbed eigenstates by χ_i, the nth state is

$$\chi_n = [\sum(l_i+1)]^{-1/2}\sum(l_j+1)^{1/2}\varphi_j. \tag{9}$$

Therefore, this state is pushed up through coherent effects from all of the degenerate levels. Since a large number of particle-hole states participate and since each term in the sum in Eq. (8) is of the order of one Mev, the nth level is pushed up by several Mev. Now, the dipole transition amplitude to any one of the unperturbed states φ_i is just

$$T_i = \int \varphi_i(\vec{r}_p, \vec{r}_h)\delta(\vec{r}_p - \vec{r}_h)H_I(\vec{r}_p)d^3r_p d^3r_h$$

$$= [(l_i+1)/3]^{1/2}\int_0^\infty R_{l_i} R_{l_i+1} r^3 dr. \tag{10}$$

Assuming, again, that the radial integrals are all equal, it is clear from Eq. (9) and the orthogonality of the φ_i that the uppermost level carries, in this approximation, all of the dipole transition strength. That is, through the particle-hole interaction, the lower levels have been denuded of their dipole transition strength and this has been transferred into the uppermost level. We propose, therefore, to call this level the "dipole level."

These arguments can be generalized to cover the transitions l_i to $l_i - 1$, as well, by changing l_i to $l_i - 1$ in the relevant matrix elements of the secular matrix and in T_i. However, whereas the approximation of setting all radial integrals equal does not seem unreasonable in the transitions l_i to $l_i + 1$ where the important transi-

tions are "nodeless to nodeless" ones, it would not seem to be justified in the transitions l_i to l_i-1. However, in heavy nuclei (e.g., see Wilkinson's calculations for Sn and Pb in reference 3) almost all of the oscillator strength is contained in the transitions l_i+1.

To take account of effects due to the neutrons in the nucleus, we consider first the case of light nuclei and neglect Coulomb forces, so that isotopic spin can be considered to be a good quantum number. Then the dipole interaction Hamiltonian, after removal of center-of-mass coordinates, is

$$H_I=(e/2)(2\pi\hbar\omega)Z\tau_3, \tag{11}$$

for a nucleus with equal number of protons and neutrons; i.e., the effective charge on the proton is $e/2$, that on the neutron, $-e/2$. Since the Hamiltonian is the third component of a vector in isotopic spin space, only $T=1$ states will be formed from applying it to the $T=0$ ground state. The particle-hole states will now be

$$(2)^{-1/2}[\varphi_i{}^{+}(\vec{r}_p,\vec{r}_h)-\varphi_i{}^{-}(\vec{r}_p,\vec{r}_h)], \tag{12}$$

where the + indicates that the first part of the wave function refers to a proton particle-hole state, and the - indicates that the other part refers to a neutron particle-hole state. In the $T=0$ state, which cannot be formed by absorption of the gamma ray, the - sign would be replaced by a + sign.

Now the question of what happens to the $T=0$ and $T=1$ levels depends on the isotopic spin dependence of the force. A repulsive particle-hole interaction containing no isotopic spin dependence will push the $T=0$ level up in energy, leaving the $T=1$ level unchanged, whereas a force of character $\vec{\tau}_i\cdot\vec{\tau}_j$ which is repulsive for like particles will push the $T=1$ level up. The Rosenfeld mixture used by Elliott and Flowers[8] is of the latter type. A more detailed discussion of effects of a force such as the Rosenfeld mixture would require introduction of spin, but there is no doubt that the isotopic spin dependence is such as to push the $T=1$ levels up.

In heavy nuclei, where isotopic spin is no longer a good quantum number, neutron and proton excitations can be treated independently. Whereas it is true that the neutron particle-hole states are created with opposite phase from the proton particle-hole ones because of the τ_3 in the interaction Hamiltonian, a force of the $\vec{\tau}_i\cdot\vec{\tau}_j$ character will give opposite signs when evaluated between particle-hole states of like particles and particle-hole states of unlike particles so that the matrix elements in the secular matrix will again tend to be all positive.

We have made radical approximations in this schematic model, but the qualitative features should be given correctly, at least for heavy nuclei. Unfortunately, detailed calculations without our simplifying assumptions are difficult to make here, and these have been carried out only for the case of O^{16}. Here, the work of Elliott and Flowers does show that the dipole transitions occur at a high energy as a result of the particle-hole interactions, but the dipole strength is large not only in the top level, but in both of the two highest levels, although the lower levels are denuded as we would predict. Nothing in our model would predict that two levels should be pushed up.

In this case of a light nucleus, there are, however, several features which are not at all well described in our picture, and may be responsible for this. First, spin-flip transitions are relatively important here, because of the low l involved, and the unperturbed energies ϵ_i for these transitions are quite different from those of the non-spin-flip transitions. (We have neglected spin in our model, but it is clear that when generalized to include spin, it will describe only the non-spin-flip transitions well, because only these are nearly degenerate in energy.) Furthermore, the transitions l_i to l_i-1 are relatively important in oxygen. Neither of these features is present in the medium and heavy weight nuclei, where almost all of the oscillator strength is carried by the l_i to l_i+1 transitions involving no spin flip.

Some general qualitative features come out of the work of Elliott and Flowers which are present in our model. In particular, it is true that the state which is pushed highest up in energy is the most symmetric state, in terms of the particle and hole, i.e., that is the state in which expansion of the χ_i in terms of the φ_i such as in Eq. (10) involves mostly + signs with our choice of phases for the φ_i. Since the matrix elements of the secular matrix are all positive, clearly this state will lie highest in energy. This is analogous to the most symmetric state lying lowest in energy when attractive forces are present, as was noted long ago by Wigner and collaborators.[9] The more symmetric the state is, the more dipole strength it carries.

We believe that our schematic model indicates

that an increasing regularity will occur in going towards heavier nuclei, so that the coherent effects can become strong enough to shift the dipole transitions up several Mev. The schematic model is, of course, no substitute for detailed calculations, but indicates the possibility of these coherent effects in a simple way.

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ANGULAR DISTRIBUTION OF NEUTRONS FOLLOWING THE NUCLEAR CAPTURE OF POLARIZED MUONS

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Measurements[1,2] of the total capture rates of negative muons in complex nuclei have indicated that the magnitudes of the squares of the coupling constants must be about the same size as those in the processes of β and μ decay. Previous results have not shown, however, whether parity conservation is violated in the interaction, and, until recently,[3] no evidence has existed concerning the relative signs of the coupling constants.

In order to examine some of these problems the authors have attempted to measure the angular distribution of the neutron about the direction of muon spin in the process

$$\mu^- + p \rightarrow n + \nu.$$

An asymmetry in this distribution would provide clear evidence of parity nonconservation. In addition, various authors[4-7] have shown that the value of the asymmetry coefficient α in the expression for the angular distribution,

$$D(\theta) = 1 + \alpha\, \vec{\sigma}\cdot\vec{p}/|\vec{p}|, \qquad (1)$$

is dependent upon the relative signs and magnitudes of the coupling constants (here $\vec{\sigma}$ represents the muon spin vector, and $\vec{p}$ the neutron momentum).

The experimental arrangement is shown in Fig. 1. The negative meson beam, of momentum 190 Mev/c, from the Liverpool synchrocyclotron was moderated by an appropriate amount of polythene so that the muons were brought to rest in the target. Pions stopped in the absorber between counters 2 and 3. The target was S^{32}; this was chosen in order to provide a sufficient number of neutrons, and to avoid complete depolarization of the muon beam.[8]

The arrival of a muon was signalled by the coincidence sequence $23\bar{4}$, and the emission of a neutron by $\bar{1}\,3\,\bar{4}\,5$ (referred to hereafter as "start" and "stop" events, respectively). Counter 5 was of a type developed by Brooks,[9] and was used in order to discriminate against the γ rays emitted following muon capture. This discrimination is essential, since the rate of emission of γ rays is comparable to that of neutrons.[2] The counter had an average efficiency for the detection of neutrons in the energy range of 4 - 15 Mev of

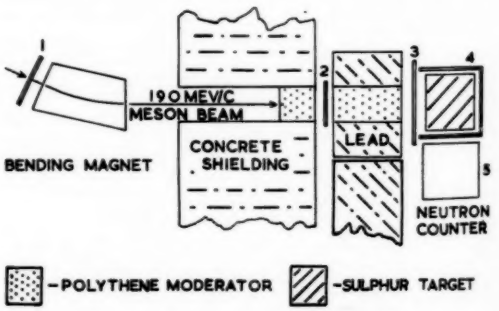

FIG. 1. Experimental arrangement.

~10%, and for γ rays up to 5 Mev of <0.5%.

The time interval between the start and stop pulses was measured by an analog converter (timesorter), which delivered pulses whose height was proportional to the time interval, to a 180-channel pulse-height analyzer (kicksorter). The stop pulses were delayed by 0.6 μsec with respect to the start pulses, thus the pulse heights corresponding to the first 0.6 μsec provided a direct measurement of the occurrence of random events and were displayed in the early channels of the kicksorter pattern.

A magnetic field was applied perpendicular to the plane defined by the centers of counters 2, 3, and 5, and caused the muon spin to precess in this plane. In this way any asymmetry in the direction of emission of the neutrons about the spin direction of the muons would be observed as a sinusoidal modulation of the characteristic muon decay curve for S^{32}, and it would yield a pattern on the pulse-height analyzer of the form

$$F(t) = N_0 e^{-t/T}[1 + \alpha\phi P \cos(\omega t + \delta)] + B. \qquad (2)$$

Here T represents the muon lifetime in S^{32}, P the muon polarization before capture, δ a phase factor, $\omega = geH/2mc$ (the symbols have the usual significance), and B is the random background. ϕ is a factor due to the geometry of the apparatus and the presence of evaporation neutrons. In the conditions of the experiment, N_0/B was found to be 2.7.

In order to reduce the background from undesirable sequences of start and stop events, the following precautions were taken. Electrons were present as a contamination in the meson beam. The pulses produced by them in counter 3 were smaller than those from muons, and were therefore rejected by a pulse-height discriminator in the starts channel. In the stops channel all pulses from counter 3 were accepted; counters 3 and 4 together provided 4π geometry in which to veto electrons from muons which decay.

In order to avoid most of the evaporation neutrons, which would yield an isotropic angular distribution, a pulse-height discriminator was used in counter 5. In this way neutrons were rejected when their recoil protons dissipated less than 3 Mev in the scintillator.

An analysis of the experimental data gave a value of 0.46 ± 0.05 μsec for T; this figure is in agreement with existing values.

In Fig. 2 the experimental results are displayed in the form $[F(t) - B]e^{t/T}$. A weighted least-squares fit to the data yielded values of $-(4.5 \pm 1.5)\%$ for the quantity $\alpha\phi P$ of Eq. (2) and $+(4^\circ \pm 19^\circ)$ for δ (in determining δ allowance has been made for the position of counter 5 with respect to the muon beam). The polarization P may be determined by measuring the asymmetry of the decay electrons; the results of the experiment described in reference 8 yield a value of $P(= 15 \pm 4\%)$ for this quantity, and thus the term $\alpha\phi = -(0.30 \pm 0.12)$. Since the quantity ϕ must be less than unity this figure represents a lower limit for the value of α. The contribution to ϕ arising from the geometry of the apparatus involves a negligible correction factor (0.99); that due to the presence of the evaporation neutrons is larger and more difficult to fix. A rough estimate for the latter effect has been made using the calculated data of reference 4, and the parameters for neutron evaporation given by Lang and Le Couteur[10]; this yielded a figure of ~0.7. Thus the quantity α is ~ -0.4.

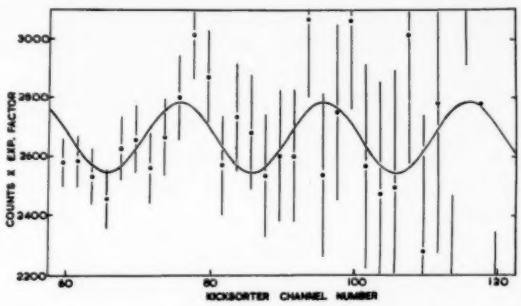

FIG. 2. Plot of the experimental points $[F(t) - B]e^{t/T}$ [see Eq. (2)] versus time t (kicksorter channel). The solid line represents the fitted curve. The kicksorter channels not displayed (prior to No. 60) contain data on the background for 0.6 μsec before zero time, together with a short "blurred" region (~0.4 μsec) about the position of zero time.

Since the error is large this result must be treated with some caution. A more accurate experiment is in progress. The result does suggest, however, that parity is not conserved in the muon capture process. Furthermore, since the helicity of the negative muon is almost certainly positive,[11] a comparison of the negative sign and the magnitude of the measured asymmetry coefficient with the theoretical forms for α suggests that there could be some enhancement of the Gamow-Teller part of the coupling constant, either through renormalization or Pauli effects.

Alternatively the large negative value for α could arise through an interference effect, if the signs of the Gamow-Teller and induced pseudoscalar coupling terms in the interaction Hamiltonian were the same. The last conclusion is supported by the theory of Goldberger and Treiman,[12] and by the analysis of Primakoff[5] of the capture rate of muons in C^{12}.

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PION MASS MEASUREMENTS USING NEUTRON TIME-OF-FLIGHT TECHNIQUES*

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Measurements of the speeds of neutrons emitted in the capture at rest of π^- mesons in hydrogen provide an excellent method for the determination of mass values for the π^- and π^0 mesons. Using energy balance of the reactions

$$\pi^- + p \rightarrow \gamma + n \text{ (8.8 Mev)} \quad (1)$$

$$\pi^- + p \rightarrow \pi^0 + n \text{ (400 kev)}, \quad (2)$$

we can deduce the mass of the π^- through Reaction (1) and the π^-, π^0 mass difference through Reaction (2). Results obtained by using time-of-flight techniques to measure the neutrons' speeds have already been reported by Gettner et al.[1] and Hillman et al.[2] for flight paths of 2 to 5 ft. This Letter reports preliminary results of our measurements for flight paths of 12.44 ft and 17.50 ft with similar techniques.

Figure 1 is a simplified schematic diagram of the experimental setup and electronics. The 110-Mev pions produced by an internal target of the 184-in. cyclotron were collimated into a 2-in.-high by 8-in.-wide beam; slowed down in carbon; counted by a 1-in.-thick plastic scintillator (π counter), 1.5 in. high by 8 in. long; and stopped in a liquid hydrogen target. The hydrogen flask was a horizontal cylinder 1 in. high and 12 in. in diameter. Neutrons, which travel downward, were detected in a plastic scintillator 18 in. in diameter, 1 in. thick, viewed by seven RCA 7046 photomultipliers, through a 1-in. Lucite light pipe. Photons, from Reaction (1) and π^0 decay, which go upward, convert in a 1/4-in.-thick Pb sheet, 20 in. in diameter, 10 in. from the center of the target, and are detected by a coincidence between two plastic scintillators, each 3/8 in. thick (γ counters).

The electronics (see Fig. 1) is composed of two fast coincidences followed by a slow coincidence. One coincidence is between the π and γ counters and another is between the dynode signals of two sets of tubes of the neutron counter—the latter coincidence mode to reduce

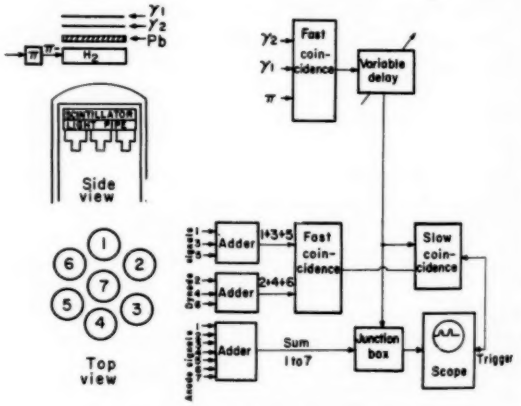

FIG. 1. Experimental arrangement.

noise accidentals. Finally a π-γ-n slow coincidence is made with 200-mμsec resolving time which triggers a 517A Tektronix scope with a 20-mμsec/cm sweep speed. The anode signals of the neutron counter tubes are combined in a distributed adding circuit and, together with a π-γ coincidence signal, are displayed on the scope sweep, of which photographs are taken. Alternate runs were made with the variable delay set for π-γ-γ and π-γ-n coincidences.

We used styrofoam delay lines for most of the inserted delay. RG-63/U delay lines were used for fine adjustment. For such long cables one must know not only the transit time of the cable but the effect of dispersion as well. A cable measuring system is used which minimizes systematic errors due to dispersion. The output of a signal generator drives a pulse shaper which puts out two pulses with the same rise time as the phototubes. The shaped pulses are fed into the vertical plates of a 5-mμsec/cm two-beam scope, one signal coming in directly and the other via the unknown cable. A third pulse derived from the pulse shaper goes to the scope trigger. The frequency of the oscillator is adjusted so that the half-heights of the succeeding pulses coincide. When this happens the period of the oscillator is the same as the length of the cables, i.e., $L/v = 1/f$. The frequency is measured by a Hewlett-Packard 524B frequency meter. In practice we measure the length of the unknown plus a fixed cable, then remove the unknown and measure the fixed cable. The difference between the oscillator period for the two measurements is the delay through the unknown. Dispersion effects are eliminated if the output pulse of the fixed cable is identical to a phototube pulse.

To analyze the time-of-flight data, we measured the distance between half heights of the two photographed pulses along with the amplitude of the signal from the neutron counter. We plotted the pulse-height spectrum of the background and compared it with that in the region of the neutron peak. We chose a pulse-height interval with the best signal-to-noise ratio, and measured the time distribution of the pulse in this interval for both the neutrons and the γ rays. The mean of the γ-ray distribution was used as our time reference point, and corresponds to the calculated photon flight time. The neutron flight time was taken to be: the photon flight time + the inserted delay + the difference between the mean of the neutron and γ-ray peaks. A composite plot of our measurements is shown in Fig. 2.

Table I lists the results of this experiment. The errors shown are over-all errors and are largely statistical.

The values quoted here agree favorably with recent accurate determinations by other means and by other groups. The currently recommended value of the π^- mass is $(273.27 \pm 0.11)m_e$.[3] Values for the π^-, π^0 mass difference have recently been reported as $(8.8^{+0.4}_{-0.3})m_e$,[4] $(8.9 \pm 0.14)m_e$,[5] and $(9.01 \pm 0.08)m_e$.[2]

The method of computing the neutron's speed from measurement at two distances is inherently more accurate in that some systematic errors are eliminated. Scattering of the neutrons may

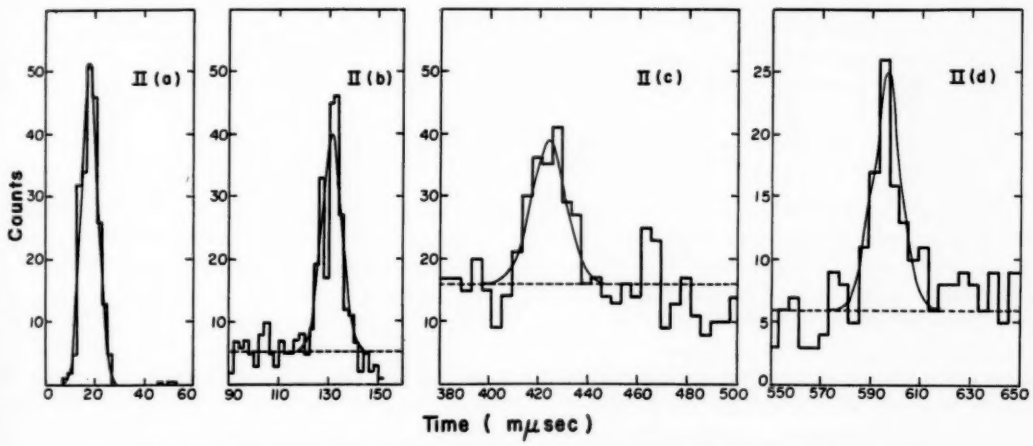

FIG. 2. Histograms of combined data. (a) γ ray at 17.50 ft; (b) 8.8-Mev neutrons at 17.50 ft; (c) 400-kev neutrons at 12.44 ft; (d) 400-kev neutrons at 17.50 ft.

Table I. Mass determinations from time of flight.

Run	Flight path (ft)	Time (mμsec)	$\pi^- - \pi^0$ mass (m_e)	π^- mass (m_e)
1	17.50 ± 0.01	130.79 ± 0.45		272.4 ± 1.1
2	12.44 ± 0.01	434.34 ± 0.79	8.984 ± 0.026	
3	17.50 ± 0.01	596.0 ± 1.1	8.998 ± 0.024	
Av(2, 3)			8.991 ± 0.020	
2 minus 3	5.055 ± 0.01	171.7 ± 1.4	9.05 ± 0.09	

shift the mean of the neutron distribution to longer times and also widen it. If the scattering is in the counter and not in the target, it will have little effect upon the value for the mass difference obtained from the difference measurement at two lengths. We have computed the scattering in the hydrogen target and found it to be negligible at these distances.

The standard deviation of the γ-ray peak is 3.5 mμsec; it is 4 to 6 mμsec for the fast neutron and 7 mμsec for the slow neutron. The size of the neutron widths is not completely understood at present and this uncertainty does not allow us to infer a meaningful limit for the π^0 lifetime.

The background in our neutron counter is currently limiting us in our ability to obtain more accurate measurements with longer flight paths. It appears to be due to three causes: radiation from the target, which decreases with solid angle and beam level; tube noise, which is constant but relatively small; and an effect that is of about constant value with depth. We are in the process of reducing the latter to allow us to increase the flight path by an appreciable factor.

We gratefully acknowledge the efforts of the many individuals who have assisted us in this experiment.

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STRANGENESS 2 MESON

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In a previous communication[1] evidence was presented for a new strange particle. The interpretation of our event assigned to this particle a strangeness -2, a negative single charge, and a mass in the neighborhood of 700 Mev. According to the conventional scheme, a meson of strangeness 2 should occur as an isotopic singlet. The particle would have $Q = +1$ and $S = +2$; the antiparticle would have $Q = -1$ and $S = -2$. We shall call them D^+ and D^-, respectively. The event discovered[2] at Dubna, which motivated our search, may constitute evidence for the D^+, while ours suggests the existence of D^-.

We wish to report some interpretations of certain other anomalous events which seem to lend support to the D-particle hypothesis. First, there are the several anomalous decays of "K" mesons both at rest and in flight. An event has been reported[3] by a Columbia group, who saw a presumed K^+ decaying at rest into a 60-Mev π^+. They interpreted this as being a decay mode $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$, and showed that 60 Mev is near the

maximum of the expected π^+ spectrum. It happens, however, that a similar event has been found[4] at Bristol, and that the π^+ energy was the same in both cases to within the experimental error. The experiment enhances the possibility that the events are examples of the decay $D^+ \to \pi^+ + K^0$. Under this assumption, they give a D^+ mass of 719 ± 2 Mev.

There are perhaps three examples of D decay in flight. One is the Dubna event. The others were two anomalous events reported[5] from Duke. Both were produced by 2.7-Bev protons in the wall of a cloud chamber. The first showed a positive particle of momentum 500 ± 25 Mev/c which decayed into a positive secondary of momentum 87 ± 5 Mev/c. The kinematics of the decay preclude the possibility that the primary particle was a K meson. From its momentum and ionization alone, the mass of the particle was estimated to be between 590 and 840 Mev. If the charged secondary is taken as π^+, without regard to the nature of the neutral secondary, the primary mass is 800 ± 50 Mev; if the decay is taken as $D^+ \to \pi^+ + K^0$, and D^+ mass is $747\,{}^{+8}_{-12}$ Mev.

The interpretation of the other Duke event is more difficult. It appeared to its discoverers to require the presence of a negative particle of approximately 750-Mev rest mass, decaying into $K^0 + \pi^-$ with subsequent decay of the K^0 into $\pi^+ + \mu^- + \nu$. This interpretation fits the kinematics of the four charged tracks that were observed, but, if the D^- particle is really present, it is not clear how it can have been produced. The primary particle in this experiment was 1.9-Bev π^-, and the mode of production suggested by the authors was of the type $\pi^- + N \to D^- +$ hyperon. But this does not conserve strangeness, and we suggest instead that they saw $\pi^- + p \to D^+ + D^- + n$, which has a threshold of 1.9 Bev.

The D particle can also be invoked for an interpretation of two anomalous interactions[6] discovered in cosmic radiation by the Wisconsin group. In each of these events, a secondary particle came to rest in the emulsion and gave rise to a well-identified K meson of about 40-Mev kinetic energy. In one of the cases, an additional short prong was observed. If these events are regarded as interactions of D^- with bound neutrons according to $D^- + n \to K^- + \Lambda^0$, and if the target neutrons are taken to be at rest, the D mass in both cases is found to be about 730 Mev.

We wish finally to report that a careful reexamination of our event has failed to make any alternative interpretation more plausible than it was at the time of our earlier Letter. In particular, one of us has made a careful study of the possibility that prong 6 of our event is a He^3 or He^4 nucleus initiating a reaction in a nucleus of the emulsion. The charged secondaries leaving the end of this track appear to be so oriented that momentum can only be conserved if at least one neutral particle accompanies them. The proton coming from the star has an energy of 3.6 Mev, and the upper limit of the energy of the incoming particle is known to be low. The only nuclear interaction which might fit these facts is $N^{15}(He^3, pn)O^{16}$ with a Q of 4.4 Mev. Analyzing the kinematics for this reaction, and taking into account only the directions of the He^3 and O^{16} momenta together with the known momentum of the proton, we find that the angle between the incident track and the resultant momentum of charged secondaries cannot exceed 35 degrees. On the other hand, the corresponding observed angle cannot be less than about 60 degrees, unless one or more of the charged particles scatters within such a small distance from the star that we cannot observe it. In view of these conclusions, and of the fact that the kinematics of the star are perfectly consistent with

Table I. Summary of possible D-meson events.

Laboratory	Present interpretation	No. events	D mass (Mev)
Columbia (em.)	$D^+ \to \pi^+ + K^0$ (rest)	1	719 ± 2
Bristol (em.)	$D^+ \to \pi^+ + K^0$ (rest)	1	719 ± 2
Dubna (b. ch.)	$D^+ \to \pi^+ + K^0$ (flight)	1	750 ± 200
Duke (d. cl. ch.)	$D^+ \to \pi^+ + K^0$ (flight)	1	$747\,{}^{+8}_{-12}$
Duke (d. cl. ch.)	$D^- \to \pi^- + K^0$ (flight)	1	750
Rochester (em.)	D^- + nucleus $\to \Sigma^+ + \Lambda^0$	1	$640\,{}^{+120}_{-100}$
Wisconsin (em.)	$D^- + n \to K^- + \Lambda^0$	2	730

well-known[7] hyperfragment mesonic decays, we feel that our earlier arguments for the interpretation of this prong as a hyperfragment have not been weakened.

Table I summarizes the present evidence for the D particle. We feel that, despite the ambiguities of the interpretation in some cases, the situation is now such that the existence of the particle cannot easily be discounted. We are therefore hoping to undertake a systematic search for the particle in a mass-separated beam from nucleon-nucleon bombardment. The early events, together with a knowledge of the mass and possible decay modes of the particle, suggest that its lifetime is comparable with that of the charged K. Its existence can therefore be confirmed by conventional techniques. If the search is successful, a direct confirmation of the properties of the particle should be attempted with a characteristic reaction such as $K^+ + p \rightarrow D^+ + \Sigma^+$.

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

PLUVINAGE METHOD FOR SYSTEMS OF THREE CHARGED PARTICLES. Larry Spruch and Martin Kelly, Physics Department, Washington Square College, New York University, New York, New York (Received June 16, 1959).

A method due to Pluvinage for finding ground-state energies of two-electron systems has been studied. His choice of a trial function, Ψ_k, is of interest for $H\Psi_k$, where H is the Hamiltonian, is free of singularities. However, his calculations had to be done numerically. It has been found possible to utilize the method in such a way that, with little loss of accuracy, the various calculations can be performed analytically. With a trial function which contains no variational parameters, the method just fails to give a bound state for H^-; the results for Z greater than 1 should be quite accurate. The introduction of a scaling factor gives a bound state. Thus, only one variational parameter is required to prove the existence of a bound state, as compared with the two parameters required for a Hylleraas type trial function. However, the calculations are much more tedious. The method has been generalized to be applicable to an arbitrary system of three charged particles which interact only through their Coulomb fields. It has been applied to the proton-proton-μ^- system, but it does not give very useful results there.

MANY-BODY PROBLEM IN QUANTUM STATISTICAL MECHANICS. III. ZERO-TEMPERATURE LIMIT FOR DILUTE HARD SPHERES. T. D. Lee, Columbia University, New York, New York, and C. N. Yang, Institute for Advanced Study, Princeton, New Jersey (Received June 17, 1959).

Using the method developed in paper I of this series, the thermodynamical properties at $T=0$ of a dilute system of hard spheres are computed for small values of ρa^3, where a is the diameter of the spheres and ρ is the particle density.

MANY-BODY PROBLEM IN QUANTUM STATISTICAL MECHANICS. IV. FORMULATION IN TERMS OF AVERAGE OCCUPATION NUMBER IN MOMENTUM SPACE. T. D. Lee, Columbia University, New York, New York, and C. N. Yang, Institute for Advanced Study, Princeton, New Jersey (Received June 26, 1959).

Starting from Rules A and B of a previous paper (I), it is shown that the grand partition function can be evaluated in terms of the statistical averages of the occupation number in momentum space. The final formulation is in terms of a simple variational principle. The procedure represents a concise and complete separation of the effect of the Bose-Einstein or Fermi-Dirac statistical character of the particles from the dynamical problem. In the case of Bose statistics, this formulation makes possible a systematic computation of all thermodynamic functions near the Bose-Einstein transition point in the gaseous phase. Applications to a system of hard spheres are discussed.

SCATTERING OF ELECTRONS BY NEUTRAL ATOMS. Bernard A. Lippmann and Marvin H. Mittleman, Lawrence Radiation Laboratory, Livermore, California, and Kenneth M. Watson, Physics Department and Lawrence Radiation Laboratory, Berkeley, California (Received June 22, 1959).

A recent reformulation of the theory of electron scattering by atoms—in which those scatterings, real or virtual, that leave the state of the atom unchanged are separated off from the remainder—has been generalized to include the effects of the Pauli principle. The case where the Hartree approximation suffices to describe the atom is considered in detail, including a calculation of the "scattering potential" to second order.

ELECTROMAGNETIC INTERACTION OF A BEAM OF CHARGED PARTICLES WITH PLASMA. Jacob Neufeld, Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received March 27, 1959).

According to Akhiezer and Fainberg there is an instability resulting from electrostatic interactions in a medium comprising an electron beam passing through plasma. This instability is associated with the growth of longitudinal

waves in the region in which the velocity of the waves is close to the velocity of the electron beam. It is shown that under similar physical conditions, i.e., in the absence of an external magnetic field, the system is stable against "transverse" electromagnetic perturbations. Therefore there is no direct mechanism for generating electromagnetic waves as a result of the interaction between the electron beam and the plasma. The only mechanism is indirect, i.e., the longitudinal waves produced by such interaction are converted into transverse waves as a result of density gradients and boundary effects.

RADIATION PRODUCED BY AN ELECTRON BEAM PASSING THROUGH A DIELECTRIC MEDIUM. Jacob Neufeld, Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received April 16, 1959).

An electron beam passing through a dielectric medium may produce an instability that is associated with the growth of longitudinal waves having a velocity close to the velocity of the beam. For a transparent dielectric medium this instability occurs if the frequency ω of these waves satisfies the following condition: $\omega_1^2 < \omega^2 < \omega_1^2 + \omega_0^2$, where ω_1 is the frequency of bound oscillators in the dielectric medium and $\omega_0 = (4\pi ne^2/m)^{1/2}$, where n is the electron density. If inhomogeneities are present these longitudinal waves may be converted into transverse waves and radiated into space. Thus, there is a possibility of a luminous effect at "Bohr frequencies" that differ from the Vavilov-Čerenkov frequencies.

DERIVATION OF HYDRODYNAMIC EQUATIONS FOR THE QUANTUM SYSTEMS OF DIATOMIC MOLECULES. Roman Zelazny, Institute of Theoretical Physics, Warsaw University, Warsaw, Poland (Received March 17, 1959).

Bogolyubov's method of derivation of the hydrodynamic equations from a quantum-statistical formalism, based on the array of distribution operators for clusters of s molecules, is adapted to the derivation of the hydrodynamic equations for a fluid composed of diatomic molecules. The general form of the hydrodynamic equations with an additional equation of angular momentum, which is coupled with the momentum equation through the antisymmetric part of the stress tensor, is obtained and all the interesting hydrodynamic quantities are calculated. A general procedure of derivation of the hydrodynamic equations by successive approximations is proposed and the equations of zeroth approximation are discussed.

GROUND-STATE ENERGY OF BOSE-EINSTEIN GAS WITH REPULSIVE INTERACTION. Katuro Sawada,* The Institute for Advanced Study, Princeton, New Jersey (Received June 5, 1959).

The explicit evaluation of the ground-state energy of a Bose gas interacting through a two-body repulsive potential as an expansion essentially in powers of the density of the particles and the scattering length is given. It is shown that the problem can be treated by using diagrammatical analysis up to any order. The energy was investigated up to the fourth order in the ordinary sense of a perturbation expansion in terms of the scattering length, but including terms up to infinite order for special classes of diagrams. The energy obtained contains in general a $\sum_n C_n \rho^n \ln\rho$ ($n \geq 2$, half-integer and integer) dependence, and the coefficient C_2 is evaluated. A self-consistent treatment is given following this analysis and is shown to lead to the formulation of Beliaev.

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FIELD OPERATORS FOR BOSONS WITH IMPENETRABLE CORES. I. EQUATIONS WHICH REPLACE THE COMMUTATION RULES. A. J. F. Siegert, Physics Department, Northwestern University, Evanston, Illinois (Received June 25, 1959).

Field operators representing particles with impenetrable cores cannot satisfy the usual commutation rules. While the customary derivation of the second-quantization formalism cannot be applied to the case of particles with nonintegrable interaction potentials, the field operators $\psi(\vec{x})$ and $\psi^\dagger(\vec{x})$ can be defined by a matrix representation which exhibits them explicitly as transformations of functions of N position vectors into functions of $N-1$ and $N+1$ position vectors, respectively. The assumption of impenetrable

cores is introduced in this definition by taking the matrix elements which lead to prohibited configurations of position vectors equal to zero. Equations which replace the usual set of commutation rules are derived from this definition for the case of hard-sphere interaction. Conversely it is shown that results which follow from the commutation rules in the standard formalism, follow from the new set of equations with the changes obviously required by the assumption of impenetrable spherical cores. For example, the operator for the number of particles in a finite domain has as eigenvalues the non-negative integers not exceeding the largest number of hard spheres which can be placed into the domain.

THEORY OF FLUORESCENCE TIME-CONSTANT MEASUREMENTS IN LIQUID AND RIGID SOLUTIONS. Hartmut Kallmann, Department of Physics, New York University, New York, New York (Received June 16, 1959).

Formulas are derived for the dependence of fluorescence upon time for the case when energy transfer from the solvent to the solute is involved. Three cases are considered: excitation by single fast particles, excitation by one burst of particles during a given period, and periodic excitation. One important point is that in the rise and decay of fluorescence from a burst of particles, it is the difference of two exponential functions and not their sum which occurs. The results are compared with the experiments of Burton and Dreeskamp, of Swank and Buck, and of Knau. These experiments are in general accordance with the derived formulas.

ELASTIC SCATTERING OF LOW-ENERGY ELECTRONS BY ARGON. B. Kivel, AVCO-Everett Research Laboratory, Everett, Massachusetts (Received June 22, 1959).

The partial-wave Schrödinger equation has been integrated numerically for a potential adjusted so that the predicted elastic scattering is in agreement with measurements by Ramsauer and Kollath. The cross section at zero energy is found to be 8×10^{-16} cm^2 which is appreciably above the minimum value of 0.3×10^{-16} cm^2 at 0.4 ev. The high value at zero energy, which results from the tail of the polarization force, will be reduced at densities of one atmosphere and above, because the field is cut off by neighboring atoms. Application of the method to predict the eigenvalues of the excited bound states of potassium indicates the validity of the static potential and that the exchange force decreases with increasing angular momentum.

OPTICAL ABSORPTION IN PURE SINGLE-CRYSTAL InSb AT 298° AND 78°K. S. W. Kurnick* and John M. Powell,† Chicago Midway Laboratories, Chicago, Illinois (Received May 11, 1959).

The absorption spectra of single-crystal homogeneous InSb were measured in the spectral range 5 to 10 microns at temperatures of 78°K and 298°K. Primary emphasis was placed on the precise determination of absorption coefficients less than 400 cm^{-1}. Absorption spectra were measured in many samples over the following range of impurity concentrations. Net impurity concentrations, expressed in atoms cm^{-3}, ranged from 5×10^{15} to 9.5×10^{16} in p-type samples, and from 2×10^{15} to 3×10^{17} in n-type samples, as determined from Hall coefficients measured at 78°K.

In general, the spectral range covered included regions where the absorption was dominated by either free-carrier absorption or valence-conduction band transitions. Free-carrier absorption in p-type InSb indicates a simple valence-band structure about $\vec{k}=0$, consisting of light- and heavy-hole bands. Free-carrier cross sections at 298°K are $\sigma_p=8.65\times10^{-16}$ cm^2 per hole and $\sigma_n=0.23\times10^{-16}$ cm^2 per electron (at 9 μ). Whereas the free hole absorption coefficient is roughly independent of wavelength, the free electron absorption σ_n varies as λ^2 and agrees well with the classical Zener-Drude model.

The main absorption edge at both temperatures may be extended to lower absorption coefficients α by subtracting the extrapolated free-carrier absorption coefficients α_c. The resultant band edge $\ln(\alpha-\alpha_c)$ values when plotted against the photoenergy ($\hbar\omega$) fit a straight line. The slopes of these band edges increase at the lower temperature and decrease (either at 78° or 298°K) as the acceptor concentration in the optical sample increases. Various models previously proposed are compared with the experimental results.

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WORK FUNCTION OF IRON SURFACES PRODUCED BY CLEAVAGE IN VACUUM. Ralph E. Simon,* Department of Physics, Cornell University, Ithaca, New York (Received June 12, 1959).

Kelvin measurements of the contact potential difference between iron surfaces prepared by cleavage in high vacuum and freshly flashed tungsten surfaces are reported. From these measurements, the work function of iron could be found since the work function of the tungsten was known. The work function of the cleavage plane of iron was 4.17 ± 0.03 electron volts at liquid nitrogen temperature. Evidence indicating that these measurements were made on clean surfaces is presented. The work function which has been determined is believed to be that of the (100) plane of iron.

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PHOTOCONDUCTIVE AND PHOTOELECTROMAGNETIC LIFETIME DETERMINATION IN PRESENCE OF TRAPPING. I. SMALL SIGNALS. A. Amith, RCA Laboratories, Princeton, New Jersey (Received March 25, 1959; revised manuscript received July 8, 1959).

Impurities which are located in the forbidden energy gap of a semiconductor are classified as traps or as recombination and generation levels, according to their capture cross sections and their proximity to the quasi-Fermi levels of the carriers. Their influence upon the photoconductance and the photoelectromagnetic effect is considered; noted in particular are their effects on the values of carrier lifetimes deduced from these photosignals. The lifetimes deduced are always too high for that type carrier of which some are trapped, and they are too small for the other type carrier. In extrinsic material the photoeffects are augmented by trapping of minority carriers, and diminished by trapping of majority carriers. In general the effects of minority carrier trapping are more severe than those of majority carrier trapping. Furthermore, the photoelectromagnetic effect is much less sensitive to trapping than is the photoconductance and may often yield the correct minority carrier lifetime in extrinsic materials. Consequently, the method of deducing carrier lifetimes by combining the photoconductance and the photoelectromagnetic effects may lead to very misleading results indeed. Yet separate measurements of the two effects over a range of temperatures will yield the carrier lifetimes, the energy level of the traps, and their density. Moreover, concomitant measurements of the spectral dependence of photoconductance and the photoelectromagnetic effect in an extrinsic semiconductor would serve to classify the impurity centers which are found, because an impurity photoelectromagnetic effect occurs only if the carriers generated from the centers are minority carriers.

EXCITON SPECTRUM OF CADMIUM SULFIDE. D. G. Thomas and J. J. Hopfield, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 10, 1959).

The reflectance and fluorescent spectra of hexagonal CdS crystals have been measured at 77° and 4.2°K using polarized light in the region of 5000 A. Structure not previously reported has been found in the reflectivity curves which leads to the identification of three exciton series. These can be understood in terms of the splitting of the valence band into three levels at $\vec{k}=0$. The observation of excited exciton states and the polarization properties of the excitons make possible (1) the determination of two of the three exciton binding energies; (2) the determination of the energy splittings of the three valence bands; (3) the verification of the symmetry assignments of the valence and conduction bands; and (4) correlation of the work of others with the present work, showing that the definite intrinsic effects are consistent both with our observations and our interpretations. The fluorescent experiments strongly suggest that the radiative decay of excitons occurs not directly, but from localized impurity exciton states in agreement with theory.

MAGNETIC PROPERTIES OF THE MIXED GARNETS $(3-x)Y_2O_3 \cdot xGd_2O_3 \cdot 5Fe_2O_3$. Elmer E. Anderson, J. Richard Cunningham, Jr., and G. E. McDuffie, U. S. Naval Ordnance Laboratory, White Oak, Maryland (Received June 9, 1959).

Polycrystalline garnets of the form $(3-x)Y_2O_3 \cdot xGd_2O_3 \cdot 5Fe_2O_3$ have been prepared for several values of x ranging from 0 to 3. Lattice constants vary linearly from 12.374 ± 0.005 A for yttrium-

iron garnet ($x=0$) to 12.463 ± 0.005 A for gadolinium-iron garnet ($x=3$). Magnetic moments were measured from 77°K to 580°K. For $0<x<0.73$, the magnetization decreases as x increases, the decrease being more pronounced at low temperatures. At $x \sim 0.73$, a compensation point occurs near absolute zero. This compensation point appears at increasingly higher temperatures as x increases beyond 0.73, until it reaches ~287°K for $x=3$. The magnetic moments for the members of the series show reasonable agreement with values calculated on the basis of the Néel theory.

FINE STRUCTURE IN THE DECLINE OF THE FERROMAGNETIC RESONANCE ABSORPTION WITH INCREASING POWER LEVEL. Ernst Schlömann, Research Division, Raytheon Company, Waltham, Massachusetts (Received June 26, 1959).

A theory of ferromagnetic resonance at high signal powers is developed. The stationary response at high power levels is investigated for the case in which the unstable spin waves have the same frequency as the applied signal. It is found that a fine structure should be superimposed on the general decline of the resonance absorption with increasing power level. This fine structure arises from the discrete nature of the spin-wave spectrum. It should be observable even if the frequency separation of adjacent spin-wave modes is much smaller than the inverse of their relaxation times. The fine structure appears as a series of kinks superimposed on the general decline of the resonance absorption with increasing power level. The separation of subsequent kinks increases with decreasing sample volume and increasing exchange field. An interpretation of experimental data along the lines suggested in this paper should yield information about the strength of the exchange coupling.

NORDHEIM'S THEORY OF THE RESISTIVITY OF ALLOYS. George L. Hall, RIAS, Baltimore, Maryland (Received June 8, 1959).

Nordheim's theory is extended to account for (1) ionic potentials that extend outside the unit cell, and (2) order of any range. It is shown that the original theory is less approximative than is generally asserted. As an incidental result, it is also shown that Flinn's electronic theory of order can be extended appreciably.

MILLIMETER-WAVE ABSORPTION IN SUPERCONDUCTING ALUMINUM. I. TEMPERATURE DEPENDENCE OF THE ENERGY GAP. Manfred A. Biondi and M. P. Garfunkel,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received June 24, 1959).

Measurements of the temperature dependence of the microwave absorption in superconducting aluminum have been made in the wavelength region 20 mm to 3 mm. The results, when plotted as isotherms of surface resistance ratio vs photon energy, show that, at a well-defined energy for each temperature, there is a rapid rise in absorption with increasing energy. This has been interpreted as the onset of absorption resulting from direct excitation of electrons across a forbidden energy gap. The isotherms then permit the determination of the temperature dependence of this energy gap. At absolute zero the gap value is found to be $\epsilon_g(0) = (3.2 \pm 0.1) k T_c$ (where $T_c = 1.178$°K is the superconducting transition temperature), in reasonable agreement with the theoretical value of $3.52 k T_c$ obtained by Bardeen, Cooper, and Schrieffer. Furthermore, the temperature variation of the gap exhibits the same shape as that given by the theory. Finally, reasonable agreement is obtained between theory and experiment concerning the detailed shapes of the surface resistance vs temperature curves over the measured wavelength range, provided that the experimental value of $\epsilon_g(0)$ is used in the theory.

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MILLIMETER-WAVE ABSORPTION IN SUPERCONDUCTING ALUMINUM. II. CALCULATION OF THE SKIN DEPTH. Manfred A. Biondi, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, and M. P. Garfunkel, University of Pittsburgh, Pittsburgh, Pennsylvania (Received June 24, 1959).

The skin depth in superconducting aluminum is calculated from the measured frequency dependence of the surface resistance through the Kronig-Kramers integral transforms. At absolute zero, it is found that the skin depth δ is in-

dependent of frequency at low frequencies but begins to increase at higher frequencies. The maximum rate of increase of δ occurs when the photon energy equals the gap energy, $h\nu = 3.2kT_c = \epsilon_g$; at this point $\delta(h\nu=\epsilon_g)/\delta(h\nu=0) \approx 1.12$, in disagreement with the calculation of Abrikosov, Gor'kov, and Khalatnikov. The maximum value of δ occurs at $h\nu \approx 4kT_c$. The superconducting penetration depth λ[i.e., $\delta(h\nu=0)$] is found to vary approximately as $\lambda(t) = \lambda(0)[1-t^4]^{-1/2}$, with $\lambda(0) = 5.15\times10^{-6}$ cm and $t \equiv T/T_c$. The effects of changes in the skin depth have been eliminated from the determination of the energy gap by calculation of the real part of the complex conductivity, σ_1. The energy gap values deduced from the behavior of σ_1 differ only slightly from the results obtained directly from the surface resistance measurements.

FERMI SURFACE IN ALUMINUM. Walter A. Harrison, General Electric Research Laboratory, Schenectady, New York (Received June 15, 1959).

The band structure of aluminum is reconsidered in a combined experimental and theoretical approach very similar to that originally used by Heine. A more careful analysis of the de Haas–van Alphen data of Gunnersen has indicated a considerable flexibility in models consistent with it and has allowed the proposal of a Fermi surface which is much closer to that expected on theoretical grounds than the model suggested by Heine. It is found here that the first Brillouin zone is completely filled; that the second zone contains a single closed surface surrounding a region of holes; and that the third zone contains a multiply-connected surface which gives rise to all of the observed de Haas–van Alphen oscillations.

THERMOELECTRIC POWER OF COLD-ROLLED PURE COPPER. R. H. Kropschot* and F. J. Blatt, Physics Department, Michigan State University, East Lansing, Michigan (Received June 15, 1959).

The difference between the thermoelectric power of severely cold-rolled and well annealed pure copper has been measured between about 8°K and 320°K. The absolute thermoelectric powers of our annealed and cold-worked samples were determined over the same temperature range by measuring the thermoelectric power of a thermocouple formed from annealed pure copper and pure lead. The thermoelectric power of cold-rolled copper is positive relative to annealed pure copper over the entire temperature range, and the effect of cold work is largest at very low temperatures where the thermoelectric power of annealed copper displays a pronounced minimum. Our results are in fair agreement with recent work by Powell and by Van Ooijen.

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ELECTRON EMISSION FROM SILICON p-n JUNCTIONS. B. Senitzky, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 17, 1959).

Electron emission from uncoated, reverse-biased, silicon p-n junctions has been investigated. A junction with a diameter of 1 cm and a reverse bias of 0.1 amp gives rise to an emission current, around its perimeter, of the order of 10^{-13} amp. The emission commences at fields which are considerably lower than those required for breakdown and is dependent on the junction bias conditions as well as the lattice temperature. The degree of dependence on the lattice temperature is a function of the bias conditions. A simple mechanism is proposed to explain these phenomena.

MICROWAVE FARADAY ROTATION IN ANTIFERROMAGNETIC MnF_2. A. M. Portis and Dale Teaney, Department of Physics, University of California, Berkeley, California (Received June 17, 1959).

Microwave Faraday rotation has been observed in MnF_2 from 30°K up to 300°K. The antiferromagnetic resonance frequency is computed from the rotation measurements below the Néel point. The results are in good agreement with direct observation of the antiferromagnetic resonance and molecular field theory extrapolations except close to the transition where evidence for magnetic clustering is found. The Kramers-Kronig relations are applied to antiferromagnetic media and it is shown that the Faraday rotation depends both on the shift of the resonance with field and on very small field-induced intensity changes which are present. A study of Faraday ellipticity

establishes the need for using terms in the equation of motion which relax the magnetization toward the instantaneous field direction.

INTERMEDIATE STATE OF HARD SUPERCONDUCTORS. A. L. Schawlow and G. E. Devlin, Bell Telephone Laboratories, Murray Hill, New Jersey, and J. K. Hulm, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received June 10, 1959).

The very large domains found in the intermediate state of hard superconductors are investigated. It is found that pure, annealed rhenium shows fine domains like other soft superconductors, but that coarse domains can be produced by cold-working the sample. Thus the large domains are not due to peculiar electronic properties of the hard superconductors, but to locally strained or impure boundary regions.

SPIN ALIGNMENT IN THE SUPERCONDUCTING STATE. P. W. Anderson and H. Suhl, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 25, 1959).

It is argued that spin alignment can and will occur for ion-core spins in superconductors, but that the alignment is in the form of extremely small domains. Central to the argument is the concept of the nonlocal susceptibility $\chi(\vec{r}-\vec{r}')$, which leads to a positive short-range Kittel-Ruderman-Yosida interaction of ion-core spins, but a negative long-range interaction of range ξ_0.

Very general arguments suggest that purely ferromagnetic alignment should not be observed in preference to this domain-like "cryptoferromagnetic" alignment.

NATURE OF DEFECTS ARISING FROM FAST NEUTRON IRRADIATION OF SILICON SINGLE CRYSTALS. Rohn Truell, Metals Research Laboratory, Brown University, Providence, Rhode Island (Received September 23, 1958; revised manuscript received July 31, 1959).

A method is described for determining the range of sizes that damaged regions created by fast neutrons may have in a silicon single crystal. In particular the use of precision ultrasonic velocity and attenuation measurements for compressional waves together with certain new results from scattering theory permit the determination of an upper and lower limit on the size of a damaged region. The lower limit is determined from the fractional velocity change arising from the irradiation and the upper limit is found from the corresponding attenuation change—or in this case the lack of it.

Under the assumption that such regions of damage are spherical, the limits obtained lie in a range from 0.01μ to 0.27μ for the radius of the region in question.

CRYSTAL POTENTIAL AND ENERGY BANDS OF SEMICONDUCTORS. I. SELF-CONSISTENT CALCULATIONS FOR DIAMOND. Leonard Kleinman and James C. Phillips, Department of Physics, University of California, Berkeley, California (Received June 17, 1959).

Approximate self-consistent potentials are constructed for diamond, first with exchange ignored, and then with exchange included according to the Slater free-electron approximation and according to a refined momentum-dependent free-electron approximation. The Hartree charge densities and energy gap are in fair agreement with experiment. Inclusion of valence exchange by the Slater approximation shows that Herman's earlier calculation was nearly self-consistent in this approximation. Agreement with experiment on charge densities and energy gap is greatly improved in comparison with the Hartree results. Further inclusion of the momentum dependence of the exchange potential does not greatly improve the charge densities and the energy gap but does alter the valence band width.

CORRELATION EFFECTS IN IMPURITY DIFFUSION. John R. Manning, National Bureau of Standards, Washington, D. C. (Received June 26, 1959).

Expressions are developed giving the correlation factor for an impurity diffusing in an otherwise pure lattice. These expressions will apply in general since, in the derivation, there is no requirement that there must be a large binding energy tending to create vacancy-impurity complexes. When this requirement is eliminated, accurate expressions can be obtained even for

the non-close-packed lattices. Both vacancy and interstitialcy mechanisms are treated. A matrix method developed by LeClaire and Lidiard for the face-centered cubic lattice is applied to various lattices to obtain approximate expressions for the correlation factor. Then a correction is applied giving the effect of those dissociating vacancies (or interstitialcies) which return to the impurity. It is found that the effect is the same as if only a fraction of the dissociative jumps actually occurred. If it is assumed that the effect of an impurity is quite localized, the probability of return can be calculated in a very straightforward manner. It is shown that the effect of returning vacancies (or interstitialcies) can be quite important. The resulting expressions are used to estimate the correlation factor in nondilute alloys.

TEST OF SPIN HAMILTONIAN FOR IRON^{3+} IN STRONTIUM TITANATE. Sol Aisenberg and H. Statz, Research Division, Raytheon Company, Waltham, Massachusetts, and G. F. Koster, Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received June 18, 1959).

The applicability of a conventional spin Hamiltonian to the paramagnetic spectrum of Fe^{3+} in strontium titanate is investigated. The work was inspired by a paper by Müller who finds deviations from a conventional spin Hamiltonian which he attributed to covalent bonding. The spectrum is remeasured and compared with the more general theory of Koster and Statz. It is found that the conventional Hamiltonian describes the spectrum about as well, in this case, as the improved theory. The remaining discrepancies vary from crystal to crystal and are due to random distortions of the Fe^{3+} site. A rather good agreement with theory was obtained for one crystal which apparently was more nearly perfect than the other measured samples. From perturbation theory, it is concluded that the deviations from a conventional Hamiltonian should be about 0.1 Mc/sec if covalency and exchange effects can be neglected. The experimental errors in the present experiments are about 1 to 2 Mc/sec. Even though for the present example it is unnecessary to resort to the improved theory, it is shown that, even in the absence of covalency, measurable deviations from a conventional spin Hamiltonian are expected in substances where the zero-field splittings and the applied magnetic field are large.

SELF-ADJUSTMENT OF INTERNAL RADIATION FIELD TO COMPENSATE FOR LINEARLY VARYING d SPACING IN X-RAY DIFFRACTION. H. Cole and G. E. Brock, International Business Machines, Research Laboratory, Poughkeepsie, New York (Received June 26, 1959).

It is now well known from studies of "anomalous transmission" that the energy flow through a perfect crystal during symmetric Laue (transmission) diffraction is along the diffracting planes. If the planes are then fanned out, as in elastic bending, the question arises as to whether or not the radiation pattern can adapt itself to the changing d spacing so as to maintain diffraction conditions. If it does, the exit Bragg angle should differ from the entrance Bragg angle by an amount proportional to the bend. Experimental evidence is presented to show that this is indeed the case. Germanium crystals, judged to be perfect because of lack of etch pits, were used. Various anomalies in the transmitted intensities indicate that defects still exist.

CLASSICAL SPIN-CONFIGURATION STABILITY IN THE PRESENCE OF COMPETING EXCHANGE FORCES. T. A. Kaplan, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received June 29, 1959).

It is pointed out that Yafet-Kittel triangular arrangements are not stable in the cubic spinel. The stability criterion used is that the classical Heisenberg energy should not decrease for small, but otherwise arbitrary, spin deviations from the configuration of interest. It is found that the Yafet-Kittel-Prince configuration can probably be stabilized by a sufficient tetragonal distortion of the pattern of B-B interactions. In addition, the classical ground state is found for the antiferromagnetic body-centered cubic with first-, second-, and third-neighbor antiferromagnetic interactions (with parameters J_1, $J_1\sigma_2$, and $J_1\sigma_3$): the spin $\vec{S}(\vec{R}_n)$ at lattice point $\vec{R}_n$ is independent of time, and is always parallel to one plane, P, the angle made by $\vec{S}(\vec{R}_n)$ with a fixed line in P being of the form $\vec{k}\cdot\vec{R}_n$ for $\vec{R}_n$ a cube corner, $\vec{k}\cdot\vec{R}_n+\pi$ for $\vec{R}_n$ a body-center position (the vector $\vec{k}$ is determined by the σ_i). The

neutron diffraction pattern for such a "spiral" configuration (with $\sigma_2 \sim 0.6$, $\sigma_3 \sim 0.1$, for example) bears a close relationship with the unusual pattern obtained by Corliss, Hastings, and Weiss with a single crystal of chromium.

EVIDENCE FOR ANTIFERROMAGNETISM IN $CoBr_2 \cdot 6H_{20}$. H. Forstat, G. Taylor, and R. D. Spence, Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan (Received June 18, 1959).

An antiferromagnetic-paramagnetic transition has been observed in a single crystal of $CoBr_2 \cdot 6H_2O$ by using the proton resonance technique and by measuring the heat capacity in the temperature range 1.65-5.05°K. The Néel temperature obtained by the magnetic measurement is 3.08°K while the heat capacity measurement gave 3.07°K. The entropy change associated with the transition was calculated from the heat capacity and yielded 1.42 cal/mole deg. The contribution to the entropy change above the Néel temperature was approximately 38% of the total entropy change, which indicates a rather slow diminution of the short-range ordering of the Co^{++} spins.

EFFECT OF SPIN-ORBIT COUPLING ON THE ENERGY LEVELS IN THE 6*d* BAND FOR ACTINIDE METALS. Guy W. Lehman, Atomics International, Canoga Park, California (Received June 24, 1959).

The importance of spin-orbit coupling in modifying the energy levels in the 6*d* band for the actinide metals is discussed. The energy matrix, H_0, used in this paper, neglecting spin-orbit effects, is derived from Kohn's variational principle by expanding the wave function in terms of the five atomic *d* orbitals having cubic symmetry. A single disposable parameter, E_d, occurs in this model such that $2E_d$ is the width of the *d* band. The spin-orbit energy matrix, H_{SO}, contains the spin-orbit splitting parameter, E_{SO}, which is equal to the $d_{3/2} - d_{5/2}$ level separation. The complete 10×10 energy matrix has been diagonalized analytically along a four-fold symmetry axis and numerically at 150 points in the Brillouin zone for the case $E_{SO}/E_d = 0.2$ appropriate to face-centered cubic thorium with $E_{SO} = 0.4$ ev and $E_d = 2$ ev. Plots of the energy variation along the two-, three-, and four-fold axes in the Brillouin zone are presented. Calculations based upon this model are also presented which show that the usual Pauli spin-paramagnetism of the conduction electrons can be modified quite significantly by spin-orbit coupling.

SOLUTIONS OF THE SECULAR DETERMINANT ON THE BRILLOUIN ZONE FACES FOR FACE-CENTERED CUBIC LATTICE VIBRATIONS. W. C. Overton, Jr., United States Naval Research Laboratory, Washington, D. C. (Received June 23, 1959).

The contours of constant frequency of the longitudinal branch have been plotted on the hexagonal Brillouin zone face to aid in the study of the vibration spectra of the two-force constant model for face-centered cubic lattices. These contours are based on machine solutions of the secular determinant as well as on closed-form solutions along the edge lines of the one-sixth portion of the hexagonal face cut by the planes of the irreducible trihedral angle (1/48 zone). Newly found closed-form solutions along the bisectrix of the one-sixth portion provide greater accuracy of the contours. The graphical study has disclosed new types of critical points on the hexagonal zone faces.

ANOMALOUS THERMIONIC EMISSION OF URANIUM MONOCARBIDE AND OTHER COMPOUNDS INVOLVING RARE EARTH AND TRANSITION ELEMENTS. E. A. Kmetko, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received June 22, 1959).

Thermionic emission constants, A^*, up to several thousand times greater than the A_0 (120 amp/cm^2 °K^2) predicted theoretically for metals have been reported for several compounds involving transition metals and rare earths with boron and carbon. It is suggested that such anomalously large emission constants, as well as some anomalously small ones, are due to the relatively large distances between the metal atoms as a result of which the energy bands originating from the incomplete atomic *f* and/or *d* sublevels are narrow enough for nondegeneracy to occur in the experimental temperature range.

PARAMAGNETIC RESONANCE SPECTRA OF CHROMIUM AND MANGANESE IN THE SPINEL STRUCTURE. R. Stahl-Brada and W. Low, Department of Physics, The Hebrew University, Jerusalem, Israel (Received June 15, 1959).

The paramagnetic resonance spectrum of Cr^{3+} was measured on the single crystal of ruby spinel $MgAl_2O_4$ at the wavelengths of 3 cm, 1.2 cm, and 8.6 mm. The spectrum confirms that Cr^{3+} is at a "B" site and can be described with an axial spin Hamiltonian $S=3/2$, $2D=0.990 \pm 0.005$ cm^{-1}, $g_{\parallel}=1.986\pm0.001$, $g_{\perp}=1.989\pm0.002$.

The paramagnetic resonance spectrum of Mn^{2+} was measured on the single crystal of $ZnAl_2O_4$ at 3 cm. The spectrum consists of six nearly isotropic lines with $A=(74.9\pm0.5)\times10^{-4}$ cm^{-1}, $g=2.000_2\pm0.001$. The cubic or axial splitting was less than 8×10^{-4} cm^{-1}. The spectrum is indicative that Mn^{2+} is located at an "A" site.

EFFECT OF HYDROSTATIC PRESSURE ON THE FERROELECTRIC PROPERTIES OF TRI-GLYCINE SULFATE AND SELENATE. F. Jona and G. Shirane, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received June 18, 1959).

The transition temperatures T_C of tri-glycine sulfate and isomorphous tri-glycine selenate are raised by the application of hydrostatic pressure. Up to 2700 atmos, the dependence of T_C on pressure is linear with a slope of 2.6×10^{-3} degree/atmos for the sulfate and 3.8×10^{-3} degree/atmos for the selenate. The Curie-Weiss law, $\epsilon_b \cong C/(T-T_C)$, is unaffected by pressure except for the shift in transition temperature. The curve of the spontaneous polarization is shifted accordingly along the temperature axis; in the vicinity of the transition point, the square of the spontaneous polarization depends linearly on temperature and pressure. The results are discussed in terms of the thermodynamic theory of ferroelectrics.

CALCULATION OF ORDER PARAMETERS IN A BINARY ALLOY BY THE MONTE CARLO METHOD. Lloyd D. Fosdick, Digital Computer Laboratory, University of Illinois, Urbana, Illinois (Received May 4, 1959).

A Monte Carlo sampling scheme similar to that used by Metropolis, Wood, and others in equations of state computations for gases has been used to investigate order-disorder phenomena in a face-centered cubic A_3B alloy. The model of the alloy assumes that the structure of the lattice is fixed and that interactions exist between first neighbors and second neighbors only. In most of the calculations detailed consideration is given to an array consisting of five unit cells on an edge (five hundred sites) with periodic boundary conditions. The long-range order and short-range order for first and second neighbors has been computed above and below the critical temperature. Using the energy parameter,

$$v_n=\tfrac{1}{2}[V_{AA}(r_n)+V_{BB}(r_n)]-V_{AB}(r_n),$$

for nth neighbors, it is found that $v_2/v_1=-0.25$ and $v_1=816$ cal/mole gives the best agreement with experiments on Cu_3Au. The critical temperature appears to vary linearly with the ratio v_2/v_1.

FINE STRUCTURE OF THE MICROWAVE SPECTRUM OF NO_2. Chun C. Lin,* Harvard University, Cambridge, Massachusetts (Received June 16, 1959).

The effects of the magnetic interactions between the electronic spin, nuclear spin, and molecular rotation on the rotational energy levels of NO_2 molecule have been investigated and a theory of the fine structure of the microwave spectrum of this molecule is presented. A new method for calculating the matrix elements of the magnetic interaction terms has been developed. Possible extension of this theory to include the general type of polyatomic molecules with $S=\frac{1}{2}$ is discussed.

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MAGNETIC MOMENT OF THE PROTON IN UNITS OF THE BOHR MAGNETON; THE MAGNETIC MOMENT OF THE ELECTRON. Sidney Liebes, Jr.,* and Peter Franken,† Department of Physics, Stanford University, Stanford, California (Received May 27, 1959).

The details of a previously reported measurement of the proton magnetic moment in units of the Bohr magneton are given. This ratio of moments, which is obtained from common mag-

netic field observations of the nuclear magnetic resonance frequency of protons in a spherical sample of mineral oil and the cyclotron frequency of free low-energy electrons, is found to be $\mu_p(\text{oil})/\mu_0 = (657.462 \pm 0.003)^{-1}$, where the uncertainty represents the estimated 50% probable error. The magnetic moment of the free proton is found, upon application of the appropriate diamagnetic correction factor, to be $\mu_p/\mu_0 = (657.442 \pm 0.003)^{-1}$. The present result may be combined with reported values for the ratio of the magnetic moment of the electron to the moment of the proton to yield for the magnetic moment of the free electron in units of the Bohr magneton,

$$\mu_e/\mu_0 = 1.001168 \pm 0.000005,$$

$$= 1 + (\alpha/2\pi) + (1.2 \pm 0.9)(\alpha^2/\pi^2),$$

where the uncertainty is the estimated 50% probable error. This result is to be compared with the current theoretically estimated value for this quantity:

$$\mu_e/\mu_0 = 1 + (\alpha/2\pi) - 0.328\,(\alpha^2/\pi^2)$$

$$= 1.0011596.$$

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VALUE OF THE RYDBERG CONSTANT. William C. Martin, National Bureau of Standards, Washington, D. C. (Received June 8, 1959).

The spectroscopic measurements of Houston and of Chu pertinent to the Rydberg constant have been adjusted to a new wavelength value (5015.6779 ± 0.0003 A in air) for the helium line used as a standard by them. This new wavelength, based primarily on recent measurements by the author and by Series and Field, replaces the value 5015.675 A accepted by Houston and Chu. The adjustment brings their data into good agreement with spectroscopic observations relative to the cadmium primary standard made by Drinkwater, Richardson, and Williams. Cohen's conclusion in 1952 that the disagreement of Houston's and Chu's results with those of Drinkwater, Richardson, and Williams was due to a discrepancy in the two standards is thus verified. In addition, a reappraisal of the data of Chu and of Drinkwater, Richardson, and Williams has resulted in a treatment of their observations substantially different from that given by Cohen. The value of the Rydberg constant resulting from a weighted averaging of all the data, $R_\infty = 109737.312 \pm 0.008\ \text{cm}^{-1}$, is, however, in excellent agreement with the value $109737.311 \pm 0.012\ \text{cm}^{-1}$ calculated by Cohen from the data of Drinkwater, Richardson, and Williams.

INITIAL ESTIMATES FOR SELF-CONSISTENT FIELD CALCULATIONS FOR ATOMS WITH LARGE ATOMIC NUMBER. Charlotte Froese, Department of Mathematics, University of British Columbia, Vancouver, British Columbia, Canada (Received March 10, 1959).

Absolute rather than interpolation methods are described for obtaining initial estimates for self-consistent field calculations with exchange. Tables have been computed so that the procedure is entirely numerical which makes it more convenient than Hartree's graphic interpolation scheme.

TRIPLET INTERVALS OF HELIUM. Gentaro Araki, Masao Ohta, and Koichi Mano, Department of Nuclear Engineering, Kyoto University, Yosida, Kyoto, Japan (Received June 12, 1959).

The triplet intervals of the deepest P state of helium are calculated by making use of the wave function recently determined by the present authors. The polarization of the s orbital, the admixture of the singlet, and the quantum-electrodynamical fourth-order correction are all taken into account. The theoretical values of the intervals are $^3P_0 - {}^3P_1 = 997.457\,11 \times 10^{-3}\ \text{cm}^{-1}$ and $^3P_1 - {}^3P_2 = 75.974\,45 \times 10^{-3}\ \text{cm}^{-1}$.

VARIATIONAL CALCULATIONS OF ENERGY AND FINE STRUCTURE FOR THE $2\,{}^3P$ STATE OF HELIUM. J. Traub and H. M. Foley, Columbia University, New York, New York (Received June 9, 1959).

Using an 18-parameter Hylleraas-type wave function containing only positive powers, a calculation has been carried out for the $2\,{}^3P$ state of helium by the Ritz variational principle. Breit's reduction was used to convert the prob-

lem from six to three variables. The energy was minimized by the approximate solution of an eigenvalue problem. With this wave function the fine structure splitting was calculated. Our best wave function yields an energy 15.5 cm^{-1} above the experimental value while the corresponding fine-structure splitting is in error by about 1 part in 10^2. All computation was carried out on an IBM-650 computer.

THEORETICAL CALCULATION OF THE BINDING ENERGY OF O^{16}. Paul Goldhammer, Physics Department, University of Nebraska, Lincoln, Nebraska (Received June 15, 1959).

The second-order perturbation procedure of Bolsterli and Feenberg is applied to the ground state of O^{16}. The two-body interaction operator employed has a Serber exchange character with repulsive core and tensor component, determined to give a reasonable fit to the properties of H^2, H^3, He^3, and He^4 to the accuracy of the perturbation procedure. The resulting eigenstate for O^{16} is found to have energy eigenvalue -129.2 Mev and rms radius 2.33×10^{-13} cm. Coulomb forces are neglected. Components in the wave function different from the zero-order shell-model state are found to have a statistical weight of about 18%.

DECAY SCHEMES OF C^{15}, N^{16}, AND O^{19}. D. E. Alburger, A. Gallmann,* and D. H. Wilkinson,† Brookhaven National Laboratory, Upton, New York (Received June 16, 1959).

An iron-free intermediate-image spectrometer has been used to measure the beta-ray spectra of 2.25-sec C^{15}, 7.4-sec N^{16}, and 29-sec O^{19} and to measure the positron-electron internal pair conversion lines occurring in C^{15} and N^{16} decays. C^{15} emits a beta-ray branch of end-point energy 9.82 ± 0.04 Mev to the $\frac{1}{2}^-$ ground state of N^{15} and a branch of 4.51 ± 0.03 Mev end-point energy. Relative intensities are $(32\pm2)\%$ ($\log ft=6.0$) and 68% ($\log ft=4.1$), respectively, and both components have the allowed shape. From pair line measurements at 1.5% resolution, the C^{15} gamma-ray energy is 5.299 ± 0.006 Mev and thus the inner beta-ray group leads to the upper member of the (5.276 - 5.305)-Mev doublet level in N^{15} known from the $N^{14}(d,p)N^{15}$ reaction. The internal pair conversion coefficient derived for the 5.299-Mev line agrees best with an $E1$ assignment. Our data require spin $\frac{1}{2}+$ or $\frac{3}{2}+$ for the 5.305-Mev level in N^{15} and spin $\frac{1}{2}+$ or $\frac{3}{2}+$ for C^{15}. Taken together with other evidence, it seems likely that both states are $\frac{1}{2}+$. No evidence could be found from gamma-ray measurements for the beta decay of C^{15} to other known states of N^{15}. Some comments are made on the intermediate-coupling model for $A=15$. In the decay of N^{16} we find that the 3.3-Mev beta-ray branch to the 7.11-Mev level in O^{16} is <11% per decay, based on the Kurie-plot analysis. A value of $(4.7\pm0.9)\%$ per decay for this branch is derived from the intensity of the 7.11-Mev pair line. O^{19} decays with beta-ray branches of 4.601 ± 0.015 Mev [$(41.5^{+2}_{-5})\%$, $\log ft=5.4$] and 3.25 ± 0.02 Mev (58.5%, $\log ft=4.5$). Other results include a value of 5.416 ± 0.015 Mev for the beta-ray end-point energy of F^{20} and a value of 6.051 ± 0.005 Mev for the energy of the pair-emitting state of O^{16}.

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Cd^{111}-Cd^{113} NUCLEAR MOMENT RATIO AND HYPERFINE ANOMALY. M. P. Klein, Lawrence Radiation Laboratory, University of California, Livermore, California, and J. S. Waugh, Department of Chemistry and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received June 26, 1959).

A double resonance method has been employed to measure the nuclear moment ratio Cd^{111}:Cd^{113}. The result is $\mu_{113}/\mu_{111}=1.046\,083\pm0.000\,003$. This value is compared with recently reported hyperfine coupling intervals. A similar experiment on Hg^{199} and Hg^{201} was not successful.

ENERGY DEPENDENCE OF FAST-NEUTRON ACTIVATION CROSS SECTIONS. A. E. Johnsrud,* M. G. Silbert,† and H. H. Barschall, University of Wisconsin, Madison, Wisconsin (Received May 1, 1959).

Fast-neutron capture cross sections of 24 nuclides ranging from $A=51$ to $A=197$ have been measured by an activation method, in the neutron energy region from 0.15 to 6.2 Mev. The neutron energy spreads were of the order of 0.1

Mev so that cross sections averaged over many energy levels of the compound nucleus were measured. Activities induced in samples by fast and thermal neutrons were compared. The relative neutron flux in the fast-and thermal-neutron activations was determined with a U^{235} fission counter. A knowledge of the energy dependence of the U^{235} fission cross section and of the thermal-neutron activation cross sections allows calculation of the fast-neutron activation cross sections.

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CALCULATIONS OF NEUTRON CAPTURE CROSS SECTIONS. C. Mossin-Kotin,* B. Margolis, and E. S. Troubetzkoy,† Columbia University, New York, New York (Received June 3, 1959).

Neutron capture cross sections are calculated for a number of target nuclei using the statistical theory of nuclear reactions. These calculations are compared with experiment. The different shapes of these capture cross sections as a function of energy are explained in terms of the effect of higher incident neutron partial waves and the competition from inelastic scattering. Using the measured capture cross sections it is seen that considerable information can be extracted concerning properties of the target and compound nuclei.

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COLLECTIVE ENHANCEMENT OF *E*2 MATRIX ELEMENTS IN LIGHT NUCLEI. S. Fallieros and R. A. Ferrell, University of Maryland, College Park, Maryland (Received June 10, 1959).

Several electric quadrupole transitions in nuclei in the neighborhood of O^{16} are discussed. The well-known strong enhancement of the single-nucleon matrix elements is interpreted as resulting from the virtual excitation of a collective 2^+ state in the O^{16} core. It is found that an energy of 18 Mev for this state gives a satisfactory account of all of the experimental data. The enhanced matrix elements are expressed in terms of an effective charge, which is calculated in detail within the framework of the nuclear shell model. The value of the effective charge depends on the particular independent-nucleon states involved in the transition and is found to be approximately 0.5, 0.7, and 0.9 for $2s$-$1d$, $1d$-$1d$, and $1p$-$1p$ matrix elements, respectively. Of special interest is the result that the N^{14} quadrupole moment should possess the relatively large value of 3×10^{-26} cm^2, or about three times the simple shell-model value. This prediction has recently been confirmed by high-energy electron scattering measurements. The relation of the present work to previous theoretical treatments of enhancement by the methods of the hydrodynamical model and of configuration mixing is discussed. The Appendix on center-of-mass effects contains an explicit demonstration of the cancelling of the classical recoil quadrupole moment of O^{17} by a quantum mechanical exchange term.

COMPOUND ELASTIC SCATTERING OF 4.2-Mev NEUTRONS IN LEAD. P. L. Okhuysen* and J. T. Prud'homme, Texas Nuclear Corporation, Austin, Texas (Received June 22, 1959).

Measurements of the differential elastic scattering cross sections of normal lead and radiolead (88% lead-206) for (4.2 ± 0.1)-Mev neutrons show a marked difference. Within experimental error this difference appears to be isotropic. It is concluded that the difference is due to compound-elastic scattering in lead-208.

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PHENOMENOLOGICAL NUCLEAR INTERACTION DERIVED FROM BINDING ENERGIES. R. Thieberger, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 8, 1959).

An attempt is made to clarify the basis of a formula for binding energies derived in the framework of the j-j coupling shell model. Studies are made of the effect of three-body correlations, by introducing a density-dependent term. It is seen that the j-j coupling calculation can be suitably corrected for the many-body effects. The hard-core influence is introduced as a pseudopotential. An attempt is made to fit the binding energies with a phenomenological central potential, including a density-dependent

term. The best exchange mixture obtained is not far from a Serber type but includes a considerable amount of Bartlett force. When the hard core is included, the best exchange mixture becomes of the Serber type. With this potential a good fit is obtained over a large range of configurations.

PROTON ALPHA-PARTICLE SCATTERING AT 20.1 Mev. J. W. Burkig,* Department of Physics, University of California, Los Angeles, California (Received May 8, 1959).

The differential scattering cross section for 20.1-Mev (laboratory energy) protons incident on helium gas has been measured. The following figures give the center-of-mass scattering angle in degrees and minutes, and the corresponding cross section (c.m.) in millibarns per steradian: 8°42′: 604; 9°57′: 335; 11°11′: 262; 12°25′: 230; 13°40′: 209; 14°55′: 204; 16°8′: 198; 18°37′: 193; 22°11′: 198; 24°51′: 195; 31°6′: 179; 37°19′: 167; 43°31′: 143; 49°45′: 122; 54°57′: 104; 87°6′: 42. At c.m. angles of 111°48′ and 161°30′ no scattered particles were detected. The counting statistics varied from about ±1% at the smallest angles to ±2% at 54°57′ (lab angle 45°0′). The estimated over-all accuracy of the above cross sections is ±4%, with the exception of the 87°6′ figure, which is accurate to about ±10%.

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SLOW-NEUTRON SCATTERING BY ROTATORS. II. Howard C. Volkin, Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio (Received June 19, 1959).

The method developed in a previous paper for extending the neutron scattering formalism of Zemach and Glauber to any type of molecular rotator have been employed to derive generalized forms of the differential cross sections for rotator scattering. A mass-ratio expansion for the treatment of the high-energy limit is illustrated on the classical cross section and then employed in the treatment of the more general quantum mechanical expression for the differential cross section. The results apply to an arbitrarily asymmetric rotator. The very low-energy approximation is carried out for the symmetric rotator, and the precedure is compared with the explicit summing of the partial cross sections for individual rotational transitions. The inelastic correction to the static approximation for interference scattering is calculated to an accuracy of first order in the mass ratios for the case of the symmetric rotator.

CROSS SECTION FOR COMPOUND-NUCLEUS FORMATION IN HEAVY-ION-INDUCED REACTIONS. T. Darrah Thomas, University of California, Lawrence Radiation Laboratory, Berkeley, California (Received June 8, 1959).

Compound-nucleus-formation cross sections for various systems of heavy ions and targets have been calculated by using two simple models. The first model, based on a square-well nuclear potential, gives reasonable agreement with experiment if a radius parameter $r_0 \cong 1.5\times10^{-13}$ cm is used. The second model, based on a diffuse nuclear potential, gives agreement if $r_0 = 1.17\times10^{-13}$ cm is used. The cross sections calculated by using the first model are presented for various energies of the systems carbon, nitrogen, oxygen, and neon incident on aluminum, potassium, copper, silver, praseodymium, gold, and uranium. Also tabulated are the average values of the orbital angular momentum for each system at each energy.

MONTE CARLO CALCULATIONS OF NUCLEAR EVAPORATION PROCESSES. III. APPLICATIONS TO LOW-ENERGY REACTIONS. I. Dostrovsky and Z. Fraenkel, The Weizmann Institute of Science, Rehovoth, Israel, and G. Friedlander, Brookhaven National Laboratory, Upton, New York, and The Weizmann Institute of Science, Rehovoth, Israel (Received June 12, 1959).

Monte Carlo calculations of nuclear reactions in the low-energy ($E < 50$ Mev) region are described. The calculations are based on the nuclear evaporation model of Weisskopf. Continuum theory was used for the calculation of inverse-reaction cross sections. In the calculation of the level densities of excited nuclei, pairing and shell energy corrections were used in terms of characteristic level displacements. The accurate equation rather than the approximate Maxwell distribution was used for the selection of the kinetic energy of the evaporated

particle. Experimentally determined Q-values for the various reactions were used. The calculations are compared with about 60 experimentally measured excitation functions of nuclear reactions in the mass range Cr^{50}-Se^{74}. Cameron's values for pairing energies were used at the outset; but a new set of pairing and shell energy correction values, which leads to substantially improved agreement with the experimental curves, is presented. The procedure which was used to arrive at this set is described and several features of this set are discussed. The need for a further downward correction of the level density of symmetrical ($A = 2Z$) nuclei is indicated. Computed excitation functions are shown for all the reactions studied as well as for several reactions for which experimental data are not yet available. Further experiments on reaction cross sections are suggested which would allow a unique determination of the pairing and shell energy corrections of level densities for any value of Z and N in the region under discussion. The existence of a unique set of these correction terms would provide strong evidence for the validity of evaporation theory.

POLARIZATION OF PROTONS IN $C^{12}(d,p)C^{13}$. R. G. Allas and F. B. Shull, Wayman Crow Laboratory, Washington University, St. Louis, Missouri (Received June 26, 1959).

The polarization of protons from the ground state $C^{12}(d,p)C^{13}$ reaction has been measured over an angular range $\theta = 15°$ to $\theta = 60°$ at a deuteron bombarding energy of 10 Mev. The polarization was found to be positive at all angles studied, the axis of quantization being defined by $\vec{n} = \vec{k}_p \times \vec{k}_d$. The observed polarizations were 18% at 15° (lab); 11% at 18°; 13.8% at 24°; 15.2% at 30°; 19.5% at 36°; 44.4% at 48°; 30% at 54°; 45.8% at 60°. The results are compared with existing polarization measurements on $C^{12}(d,p)C^{13}$.

PROTON-PROTON SCATTERING AT 10 Mev. L. H. Johnston and D. E. Young,* School of Physics, University of Minnesota, Minneapolis, Minnesota (Received June 22, 1959).

Differential cross sections have been measured for scattering of 9.69-Mev protons by hydrogen gas, covering the laboratory angular range from 45° to 5°. The angular resolution is $\pm\frac{1}{2}°$ at small angles, and estimated absolute probable errors ±0.7% except at the smallest angles. The interference minimum of 51.4 millibarns occurs at 34°(c.m.) and the 90° cross section is 54.6 mb. The data can be fitted with the following set of phase shifts: $^1S_0 = 54°$, $^3P_0 = +2.83°$, $^3P_1 = -5.07°$, $^3P_2 = +2.22°$, $^1D_2 = +0.2°$.

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PROTON-PROTON POLARIZATION AT 16 MEV. William A. Blanpied,* Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 11, 1959).

The proton-proton polarization has been measured at 16.2 ± 0.2 Mev and 25 degrees in the laboratory system, using the familiar double-scattering method, with hydrogen gas as second scatterer. Instrumental asymmetries were practically eliminated by using carbon and copper alternately as first targets. The polarization was found to be +(0.6 ± 0.5)%. Theoretical implications of the work are discussed.

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ZERO-SPIN EXCITED STATE IN Ca^{42}. I. Asplund and T. Wiedling, Department of Physics, University of Stockholm, Stockholm, Sweden (Received June 15, 1959).

An angular correlation measurement has been carried out on the 0.3 Mev -1.5 Mev gamma cascade in Ca^{42}. It is shown that the observed angular correlation function $W(\theta) = 1 + 0.33\ P_2(\cos\theta) + 1.07\ P_4(\cos\theta)$ fits the spin sequence 0 - 2 - 0.

EXCITATION FUNCTION FOR THE $Al^{27}(d, \alpha p)Na^{24}$ REACTION BETWEEN 0 AND 28.1 Mev. P. A. Lenk and R. J. Slobodrian, Synchrocyclotron Laboratory, Nuclear Physics Division, Physics Department, Comisión Nacional de Energía Atómica, Buenos Aires, Argentina (Received June 15, 1959).

The excitation function for the $Al^{27}(d, \alpha p)Na^{24}$ reaction has been measured between 0 and 28.1 Mev. The external beam facilities of the Buenos Aires 71-in. synchrocyclotron were used together with the stacked foil technique. Between 19 and

28.1 Mev, present results are more precise than those previously obtained, showing that the maximum of the cross section is located at 24.25 Mev with a value of 51.4 mb.

NUCLEAR MAGNETIC MOMENTS FROM HYPERFINE STRUCTURE DATA. N. J. Ionesco-Pallas, Rumanian Institute for Atomic Researches, Bucharest, Rumania (Received June 1, 1959).

The nuclear magnetic moments determined from the hyperfine structure of the $^2S_{1/2}$ and $^2P_{1/2}$ states are systematically smaller than those determined by methods of magnetic resonance. The Breit-Crawford-Schawlow correction, which takes into account the finite dimensions of the nucleus, together with the Bohr-Weisskopf correction which takes into account the spatial distribution of nuclear magnetism, succeeds in explaining at least the order of magnitude in the above difference. However, in the two corrections, certain factors are determined graphically, while others are to a certain extent erroneous, owing to incomplete solving of the Darwin-Gordon differential system. All these difficulties are removed in the present paper and the final result is a completely analytical expression for the total correction. The numerical calculations made for $_{80}Hg^{199}$ starting from the ground state $^2S_{1/2}$ (Hg II) are in good agreement with the value of the nuclear moment determined by magnetic resonance.

ELASTIC π^+-p AND p-p SCATTERING AT 1.23 Bev/c. L. O. Roellig* and D. A. Glaser, Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan (Received June 16, 1959).

Elastic π^+-p scattering at 1.1 Bev and elastic p-p scattering at 582 Mev have been measured using a propane bubble chamber. On the basis of 661 identified π^+-p elastic scatterings found in the scanning of 1.726×10^6 cm of pion track, the total elastic cross section is found to be 12.3 $\pm$ 1.2 mb. The differential cross section is rather isotropic at large angles and exhibits a strong peak for small forward scattering angles. If the forward peak is interpreted as diffraction scattering according to the optical model, the data are best fitted by a proton with a π^+-p interaction radius, $R=(0.99^{+0.13}_{-0.11})\times10^{-13}$ cm and an opacity, $O=0.70^{+0.06}_{-0.07}$.

The total cross section for p-p elastic scattering at 582 Mev was found to be 24.2 ± 1.6 mb on the basis of 2442 elastic scatterings observed in the scanning of 3000×10^6 cm of proton track. Both differential and total p-p cross sections are in excellent agreement with the results of counter experiments in this energy region.

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K^+-NUCLEON SCATTERING IN THE TAMM-DANCOFF APPROXIMATION. Grzegorz Bialkowski and Andrzej Jurewicz, Institute for Theoretical Physics, Warsaw University, Warsaw, Poland (Received June 23, 1959).

Calculations of K^+-nucleon scattering phase shifts are reported. A simple model of Yukawa interactions between K mesons and baryons (containing no derivatives) is adopted. The method used is a three-dimensional Tamm-Dancoff approximation but with no simplifications as far as recoil effects are concerned. This method is particularly convenient for K^+-nucleon scattering, since the principle of associated production excludes graphs leading to nonrenormalizable effects.

The resulting integral equation has been solved for the $T=1$, $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ states in the energy region up to 1.26 Bev. The coupling constant is the only parameter to be determined from experimental data. This was chosen to fit the experimental value of the total cross section at about 350 Mev. The best agreement with experiment was obtained for $(G^2+\mathcal{G}^2)/4\pi=12.5$. This value is much greater than the corresponding one adopted in previous papers.

PANOFSKY RATIO FOR NEGATIVE PIONS ABSORBED IN HYDROGEN. E. L. Koller and A. M. Sachs, Columbia University, New York, New York (Received June 11, 1959).

The ratio of mesonic to radiative capture of stopped negative pions in hydrogen, i.e., the Panofsky ratio, provides a useful link in analyzing the results of photoproduction of pions and pion-nucleon scattering. Previous measurements of the Panofsky ratio indicate possible systematic errors in the experimental techniques. The ratio has been remeasured using (a) a lead glass Čeren-

kov counter of improved resolution to measure the total energy of the gamma rays (±16% for the 129-Mev gamma rays measured in this experiment), (b) time-of-flight measurement of the 180-degree recoil neutrons to determine the shape of the pulse-height spectrum of the high-energy gamma ray, and (c) a collimator of variable dimensions to test the possible effect of degradation of the high-energy gamma rays in the collimator. The value obtained for the ratio is 1.46 ± 0.10, in good agreement with the Liverpool experiments.

LOW-ENERGY π MESONS IN THE COSMIC RADIATION. Mircea Fotino,* Department of Physics, University of California, Berkeley, California (Received May 27, 1959).

The intensity of π mesons in the atmosphere is investigated in the energy region $E_{\text{kin}} \lesssim 50$ Mev at sea level and mountain altitudes. Positive π mesons coming to rest in an absorber-scintillator are identified by the characteristic π-μ decay. They are detected electronically and recorded photographically.

The value 1.45 ± 0.09 is adopted for the negative-to-positive ratio at small energies.

The omnidirectional intensities J_{π^+} of slow positive π mesons at sea level (91 m), Echo Lake, Colorado (3260 m), and Mt. Evans, Colorado (4310 m) are found to be 0.0094 ± 0.0024, 0.098 ± 0.005, and 0.240 ± 0.011 g^{-1} day^{-1}, respectively. The altitude dependence of $J_{\pi^\pm}$, $J_{\mu^\pm}$, and $J_{\pi^\pm}/J_{\mu^\pm}$ is given and the attenuation length of slow π mesons in the atmosphere found to be $L_a^{\pi} \approx 120$-130 g/cm^2 in this altitude interval. A discussion covering most of the available results along the same line is presented.

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SEA-LEVEL COSMIC-RAY MASS SPECTRUM IN THE INTERVAL $30m_e$-$2000m_e$. G. G. Fazio,* Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts, and M. Widgoff,† Harvard University, Cambridge, Massachusetts, and Brown University, Providence, Rhode Island (Received June 25, 1959).

In nuclear emulsion stacks exposed to sea-level cosmic radiation under 180 g cm^{-2} of iron, no evidence for mass values in the regions $30m_e$ to $100m_e$ and $400m_e$ to $900m_e$ was found. If such particles do exist, their intensity relative to that of μ mesons stopping in the same range interval must be as follows:

Range of mass values	Intensity
$30m_e$ to $100m_e$	$\leq 0.13\%$
$400m_e$ to $900m_e$	$\leq 0.04\%$

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SEARCH FOR ANOMALOUS LIFETIME VALUES IN SEA-LEVEL COSMIC RADIATION. G. G. Fazio* and D. M. Ritson, Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received June 25, 1959).

By use of a scintillation counter telescope, sea-level cosmic radiation was investigated for the existence of particles of mass greater than $60m_e$, decaying with lifetimes in the millisecond region. If such particles exist, their intensity, relative to μ mesons, must be as follows:

Range of lifetimes	Intensity relative to μ mesons
10^{-4} to 10^{-2} sec	$\leq 0.005\%$,
10^{-4} to 5×10^{-2} sec	$\leq 0.03\%$,
10^{-4} to 10^{-1} sec	$\leq 0.05\%$,
10^{-4} to 10^{-1} sec	
10^{-4} to 10^{-1} sec (for decay into a light meson)	$\leq 0.01\%$,
10^{-4} to 1.0 sec	$\leq 0.5\%$.

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UPPER BOUNDS ON SCATTERING LENGTHS FOR STATIC POTENTIALS. Larry Spruch and Leonard Rosenberg, Physics Department, Washington Square College, New York University, New York, New York (Received June 22, 1959).

It is shown that in the zero-energy scattering of a particle by a center of force, where no bound state exists, the Kohn variational principle provides an upper bound on the scattering length. A bound may also be obtained from Hulthén's method, although with the same form of trial function the

Kohn result will be lower (and therefore better) than the one obtained from the Hulthén principle. The Rubinow formulation need not provide a bound; for those calculations which have been performed in this form, the results may be converted without any further calculations so that they correspond to the Kohn form, and therefore, under the circumstances considered, do give a bound. Analogous results hold for states of non-zero orbital angular momentum.

ACTIVE GRAVITATIONAL MASS. Charles W. Misner and Peter Putnam, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 22, 1959).

Tolman has stated that..."disordered radiation in the interior of a fluid sphere contributes, roughly speaking, twice as much to the gravitational field of the sphere as the same amount of energy in the form of matter." The gravitational pull exerted by a system on a distant test particle might therefore at first sight be expected to increase if within the system a pair of oppositely charged electrons annihilate to produce radiation. This apparent paradox is analyzed here in the case where gravitational effects internal to the system are unimportant. It is shown that tensions in the wall of the container compensate the effect mentioned by Tolman so that the net gravitational pull exerted by the system does not change.

GENERAL RELATIVISTIC FLUID SPHERES. H. A. Buchdahl,* Institute for Advanced Study, Princeton, New Jersey (Received June 16, 1959).

In Part I of this paper certain well-known results concerning the Schwarzschild interior solution are generalized to more general static fluid spheres in the form of inequalities comparing the boundary value of g_{44} with certain expressions involving only the mass concentration and the ratio of the central energy density to the central pressure. A minimal theorem appropriate to the relativistic domain is derived for the central pressure, corresponding to a well-known classical result. Inequalities involving the proper energy and the potential energy are also considered, as is the introduction of the physical radius in place of the coordinate radius. A singularity-free elementary algebraic solution of the field equations is presented and exact values obtained from it compared with the limits prescribed by some of the inequalities. In Part II an answer is given to the question whether the total amount of radiation emitted during the symmetrical gravitational contraction of an amount of matter whose initial energy, at complete dispersion, is W_0 can ever exceed W_0.

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EXACT THREE-VARIABLE SOLUTIONS OF THE FIELD EQUATIONS OF GENERAL RELATIVITY. B. Kent Harrison,* Palmer Physical Laboratory, Princeton, New Jersey (Received June 22, 1959).

In order to trace out with more understanding the consequences of general relativity it is advantageous to have exact solutions of Einstein's field equations which show more detail than the familiar ones with their high symmetry. In the present investigation, based on the method of separation of variables, all solutions of the field equations for empty space have been found which (1) have the "linked pair" form

$$g_{ij} = \pm \delta_{ij} A_i^2(x^0, x^1) B_i^2(x^0, x^3),$$

and which (2) are nondegenerate—so far as could be determined—in the sense that all the g_{ij} cannot be reduced to functions of only two variables. Other solutions have been obtained from the solutions of the above form by interchange of variables. Explicit expressions are given for all twenty nondegenerate solutions, all apparently new. Of degenerate solutions, ten are presented, not all of them new. All thirty solutions are examined with respect to possible physical and geometrical interpretations.

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EVALUATION OF DISPERSION RELATIONS. R. Blankenbecler, Princeton University, Princeton, New Jersey, and S. Gartenhaus, Purdue University, Lafayette, Indiana (Received June 18, 1959).

A new evaluation of the dispersion relations for mesonic phenomena is proposed. The method, which utilizes the comparison function procedure,

makes explicit use of crossing symmetry and allows for an exact treatment of nuclear recoils. For the case of meson-nucleon scattering at low energies, an expansion of a first order solution is made in inverse powers of the nucleon mass and agreement with the results of a previous evaluation is obtained. The extension of the method to other processes is briefly discussed.

PHOTOPRODUCTION OF π MESONS. S. Gartenhaus, Purdue University, Lafayette, Indiana, and R. Blankenbecler, Princeton University, Princeton, New Jersey (Received June 18, 1959).

The dispersion relations for photoproduction of mesons at moderately low energies are examined by means of the comparison function method which was proposed in an earlier paper. Assuming that only the (3, 3) state is appreciably modified by rescattering effects, an approximate solution is obtained. Nucleon recoil and crossing symmetry are treated exactly. The static limit of this solution yields substantial agreement with the results of Chew, Goldberger, Low, and Nambu. It is hoped that an evaluation including effects of nucleon recoil will improve the agreement in the resonance region.

INVISIBILITY OF THE LORENTZ CONTRACTION. James Terrell, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received June 22, 1959).

It is shown that, if the apparent directions of objects are plotted as points on a sphere surrounding the observer, the Lorentz transformation corresponds to a conformal transformation on the surface of this sphere. Thus, for sufficiently small subtended solid angle, an object will appear—optically—the same shape to all observers. A sphere will photograph with precisely the same circular outline whether stationary or in motion with respect to the camera. An object of less symmetry than a sphere, such as a meter stick, will appear, when in rapid motion with respect to an observer, to have undergone rotation, not contraction. The extent of this rotation is given by the aberration angle $(\theta-\theta')$, in which θ is the angle at which the object is seen by the observer and θ' is the angle at which the object would be seen by another observer at the same point stationary with respect to the object. Observers photographing the meter stick simultaneously from the same position will obtain precisely the same picture, except for a change in scale given by the Doppler shift ratio, irrespective of their velocity relative to the meter stick. Even if methods of measuring distance, such as stereoscopic photography, are used, the Lorentz contraction will not be visible, although correction for the finite velocity of light will reveal it to be present.

UNIFORM ELECTROMAGNETIC FIELD IN THE THEORY OF GENERAL RELATIVITY. B. Bertotti, Institute for Advanced Study, Princeton, New Jersey (Received June 22, 1959).

A cosmological solution of the Einstein-Maxwell's field equations, corresponding to the case of a uniform (that is, covariant constant) electromagnetic field, is derived by means of simple geometrical arguments; the Riemannian manifold it corresponds to is the product of two ordinary surfaces of constant curvature, whose type and radius depend on the values of the cosmological constant and the invariants of the electromagnetic field. The world lines of charged test particles have also a very simple geometrical meaning.

INTERACTION OF STATIC NUCLEONS. R. E. Cutkosky, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 17, 1959).

The Heitler-London method has been applied to the calculation of the energy of interaction of two nucleons, as given by the fixed-source model. Numerical results are also given for the normalization of the state vector and the number of mesons in the cloud, for the states which comprise the deuteron. Particular attention is given to the rate of convergence of the expansions and the influence of the excited states of the nucleons. The relation to the Tamm-Dancoff method is also discussed in detail. It is shown that in $T=0$ states, the interaction does not appear to differ in any significant way from the one-meson exchange term. In $T=1$ states, on the other hand, very large contributions are obtained from the higher-order terms. These contributions have such a nature as to suggest that even at low energies it is improper to apply the

fixed-source model, with the assumption that the meson cloud follows adiabatically the motion of the nucleons, to $T=1$ states.

STRUCTURE OF WEAK INTERACTIONS AND UNWANTED PROCESSES. N. Cabibbo and R. Gatto, Istituto di Fisica e Scuola di Perfezionamento in Fisica Nucleare dell'Università di Roma, Roma, Italia and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italia (Received June 30, 1959).

A general discussion of the forbidden reactions such as $\mu\to e+\gamma$, $\mu\to e+e+e$, $\mu+N\to e+N$, $\mu\to e+\gamma+\gamma$, etc., which would arise from a possible nonlocal structure of the weak interactions is given. It is shown, by a canonical transformation, that the dominating terms of the structure, in an expansion in terms of the inverse of the average intermediate mass, do not contribute to any forbidden reaction. A discussion of the μ-e conversion process for a bound μ is given, and the rate is calculated for a particular model.

COMPLETION AND EMBEDDING OF THE SCHWARZSCHILD SOLUTION. C. Fronsdal, Theoretical Study Division, CERN, Geneva (Received June 9, 1959).

An analytic manifold is found, the most important properties of which are that it is complete and that it contains the manifold of the Schwarzschild line element. It is thus the complete analytic extension of the latter. The manifold is represented as a Riemannian surface in a six-dimensional pseudo-Euclidean space. The subspace $d\varphi=d\theta=0$ is visualized as a two-dimensional Riemannian surface in a three-dimensional hyperplane in the six-dimensional space. Although the manifold admits groups of motion isomorphic to the real three-dimensional rotation group and the one-dimensional translation group, it is impossible to introduce a global time-coordinate in such a way that the latter is realized as translations in time. Hence in any global set of coordinates the gravitational field is nonstationary, although it can be made stationary for $r>1$ to any desired approximation. The question of what happens to small test bodies reaching the Schwarzschild critical radius is discussed.

PIONIC DECAY MODES OF LIGHT Λ-HYPERNUCLEI. R. H. Dalitz and L. Liu, The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois (Received June 19, 1959).

Decay probabilities are calculated for pionic modes of decay of the Λ hypernuclei $A\leq 5$. An effect of the Pauli principle omitted in previous calculations for the two-body π^- and π^0 modes is now included. An estimate for the total decay probability of all π^- (or π^0) modes based on the completeness relation is checked by comparison with calculations based on detailed models of the decay process for two particular systems. As a result, the present data on ${}_{\Lambda}H^4$ decay are now consistent with either (a) spin $J=0$ for any value p/s less than 1.5, or (b) spin $J=1$ for any value p/s greater than 1.2, a much weaker conclusion than that reached previously. In either case, if the $\Delta T=\frac{1}{2}$ rule is roughly correct for Λ decay, the two-body mode $\pi^0+\alpha$ will be prominent among ${}_{\Lambda}He^4$ decay modes. The π^0/π^- ratio will then be large (~1.5) for ${}_{\Lambda}He^4$ decay and small (~1/6) for ${}_{\Lambda}H^4$ decay, compared with the value ~1/2 for ${}_{\Lambda}He^5$ and ${}_{\Lambda}H^3$ decay.

CONTRIBUTION OF NUCLEON CURRENTS TO RADIATIVE MUON CAPTURE IN HYDROGEN. G. K. Manacher and L. Wolfenstein, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 15, 1959).

The observables for the reaction $\mu^-+p\to n+\nu+\gamma$ are calculated for a general $(pn\mu\nu)$ coupling including all terms of order (μ/M) and leading terms of order $(\mu/M)^2$, where μ is the meson mass and M the nucleon mass. The nucleon anomalous magnetic moments are handled by the inclusion of a simple Pauli term and all other virtual pion effects are omitted here. The (μ/M) terms change the radiative capture rate by as much as 30%. Parity-nonconserving effects are decreased only slightly.

ANALYTIC PROPERTIES OF PERTURBATION EXPANSIONS. J. C. Taylor, Imperial College of Science and Technology, London, England (Received June 29, 1959).

A method is introduced for locating and interpreting the singularities in finite-order perturbation expansions of Green's functions, in cases

where the external momenta are complex vectors with real scalar products. In particular, when the external momenta form a real Euclidean set, there is a simple graphical construction, applicable to all finite orders. As an example of a case in which the external momenta form a real set in a space of signature $(+,+;-,-)$, a feature of Mandelstam's representation for scattering amplitudes is interpreted in perturbation theory. Finally, the ranges of values of momentum-transfer for which (in perturbation theory) dispersion relations certainly hold are enlarged, in particular by using the fact that the pion is pseudoscalar.

SOME ASPECTS OF THE COVARIANT TWO-BODY PROBLEM. I. THE BOUND-STATE PROBLEM. Susumu Okubo, Department of Physics, University of Rochester, Rochester, New York, and David Feldman, Department of Physics, Brown University, Providence, Rhode Island,* and Brookhaven National Laboratory, Upton, New York (Received June 1, 1959).

A study has been made of the bound states of the Bethe-Salpeter equation for the nucleon-antinucleon system, including the ladder and pair-annihilation diagrams. For simplicity, nucleons and mesons were taken to be scalar, the latter having zero rest mass. Pair effects enter only in S states with the bound states corresponding to the poles of the meson propagator D_F'. The Bethe-Salpeter equation is closely related to the integral equation for the generalized vertex operator Γ; this has been solved by using an integral-transform method similar to that of Wick and Cutkosky, under the assumption that the nucleon mass is large compared to the binding energy. After performing a self-energy subtraction, the energy eigenvalues are found as a function of the coupling constant. These have the form given by the usual Bohr formula plus corrections. Finally, some comments are made with respect to the extension of the formalism to mesons with nonzero mass and spinor nucleons.

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SOME ASPECTS OF THE COVARIANT TWO-BODY PROBLEM. II. THE SCATTERING PROBLEM. Susumu Okubo, Department of Physics, University of Rochester, Rochester, New York, and David Feldman, Department of Physics, Brown University, Providence, Rhode Island (Received June 1, 1959).

By way of extension of a previous study of the bound-state problem within the framework of the covariant Bethe-Salpeter formalism, the scattering of nucleons by nucleons has been investigated in the ladder approximation, assuming scalar nucleons and zero-mass scalar mesons. The solution of the scattering problem can be effected by using an integral-transform method which is similar to that developed for the bound-state problem, subject to the assumption that the nucleon mass is large compared to the kinetic energy in the center-of-mass system. An essential complication which is now encountered is the appearance of an infrared divergence. Two methods of circumventing this difficulty are discussed. The cross section can be calculated from the Green's function by a limiting process; the usual S-matrix formalism leads to incorrect results in this case, and an amplitude renormalization is required. In this connection, it is instructive to re-examine, in detail, the Coulomb-scattering problem in momentum space, since this is very closely related to the Bethe-Salpeter scattering problem. The Green's-function method turns out to be unsuitable for the calculation of higher-order corrections to the nucleon-nucleon scattering cross section, so that here a cutoff procedure must be used.